# Country Size and Productivity: An analysis of heterogenous firms and differential beachhead costs

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**Abstract**

This paper modifies the heterogenous fims and trade model by Melitz (2003) by explicitly modelling the beachhead cost of a firm in a new market as a function of market size. This leads to several new predictions compared to the standard model. In particular, the productivity of non exporters and exporters depend on market size. Also, manufacturing export shares vary inversly with market size. However, export shares converge as markets are integrated. The empirical part of the paper offer support for our model specification.

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*Keywords :heterogenous firms, international trade, beachhead costs*

## 1 Introduction

It is empirically well established that there are systematic productivity differences among firms; see Tybout (2003) for a survey.[1] In particular exporting firms tend to be more productive and larger than domestic firms. Also multinational firms tend to be more productive than exporters (Helpman, Melitz and Yeaple, 2004).

These empirical results have spurred the development of a new theoretical literature on trade with heterogenous firms. The explanation for the empirical finding that exporters are more productive than domestic firms is either iceberg trade costs associated with exports, as in Bernard et. al. (2003), or higher fixed costs (beachhead cost) associated with market entry into a foreign market, as in Melitz (2003) and Yeaple (2005). Only the most productive firms

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[1]Other studies include Aw, Chung and Roberts (2000), Bernard and Jensen (1995, 1999a,b, 2001), Clerides, Lach and Tybout (1998) as well as Eaton, Kortum and Kramarz (2004).

will find it profitable to pay the extra cost necessary for exports, and export firms will therefore on average be more productive.

We ask the question whether patterns of heterogeneity across firms and differences between non-exporters and exporters vary systematically across countries. More specifically, among all possible sources of such cross country variation, we ask the question if country size matters because country size might determine the size of the beachhead cost that entering firms have to pay. We will here interpret one component of the beachhead cost as the marketing cost of introducing a new variety in a market. It is then natural that the cost of entry into a market depends on the size of the market.[2] For instance, the marketing costs of establishing a new product in a large market such as the U.S. is much higher than in a small market. That the fixed entry cost depends on the market size is normally taken for granted in the marketing literature, where the marketing cost per sales is a key variable.[3]

This paper analyses a modified version of the Melitz (2003) model, where a variable component of the cost of entry into a market depends on the size of the market. Several new results emerge. First, exporters as well as non-exporters in a large market are on average more productive than in a smaller market. It is also shown that overall productivity is higher in a large country. Second, as in Melitz (2003), exporters are more productive than non-exporters. However, the productivity premium between exporters and non exporters decreases with the home country size. Third, exporters to a large market tend to be more productive than exporters to a small market when the relative beachhead cost for exporters is high compared to that of local firms. The opposite holds *mutatis mutandis*. Fourth, the manufacturing export share decreases in the size of the exporting country - a result generally taken for granted on the aggregate level. Finally, it is shown that export shares converge as markets are integrated.

The theoretical results are supported by the empirical section of the paper, where it is shown how productivity is positively associated with market size. Also, manufacturing export shares are affected by market size in accordance with our theoretical predictions.

The paper is organised as follows: Section 2 contains the model, and section 3 presents the theoretical results. 4 contains empirical tests of our theoretical predictions. Finally, section 5 concludes.

## 2 The Model

This paper introduces a modified version of the Melitz (2003) monopolistic competition trade model with heterogeneous firms.

[2] Note that empirical implementations of the Melitz model typically allows for a country specific fixed cost (beach head cost). See e.g. Helpman, Melitz and Yeaple (2004) and Helpman, Melitz and Rubinstein (2007).

[3] See e.g. Buzzell et.al. (1975).

## 2.1 Basics

There are two countries, home and foreign (denoted by '*'), and a single primary factor of production labour, L, which produces goods in the A-sector and the M-sector. The A-sector (traditional sector) is a Walrasian, homogenous-goods sector with costless trade. The M-sector (manufactures) is characterised by increasing returns, Dixit-Stiglitz monopolistic competition and iceberg trade costs. M-sector firms face constant marginal production costs and three types of fixed costs. The first fixed cost, $F_E$, is the standard Dixit-Stiglitz cost of developing a new variety. The second and third fixed costs are 'beachhead' costs reflecting the one-time expense of introducing a new variety into a market. This cost is here assumed to depend on the size of the market.

There is heterogeneity with respect to firms' marginal costs. Each Dixit-Stiglitz firm/variety is associated with a particular labour input coefficient – denoted as $a_j$ for firm $j$. After sinking $F_E$ units of labour in the product innovation process the firm is randomly assigned an '$a_j$' from a probability distribution G(a).

Our analysis focuses exclusively on steady state equilibria and intertemporal discounting is ignored; the present value of firms is kept finite by assuming firms face a constant Poisson hazard $\delta$ of 'death'.

Consumers in each nation have two-tier utility functions with the upper tier (Cobb-Douglas) determining the consumer's division of expenditure among the sectors and the second tier (CES) dictating the consumer's preferences over the various differentiated varieties within the M-sector.

All individuals in country $k$ have the utility function

$$U_k = C_M^{\mu} C_A^{1-\mu}, \tag{1}$$

where $k = H, F$, $\mu \in (0,1)$, and $C_A$ is consumption of the homogenous good. Manufactures enter the utility function through the index $C_M$, defined by

$$C_M = \left[ \int_0^n c_i^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}, \tag{2}$$

$n$ being the mass of varieties consumed, $c_i$ the amount of variety $i$ consumed, and $\sigma > 1$ the elasticity of substitution.

Each consumer spends a share $\mu$ of his income on manufactures, and demand for a domestically produced variety $i$ in the home country is therefore

$$x_i = \frac{p_i^{-\sigma}}{P^{1-\sigma}} \mu Y, \tag{3}$$

where $p_i$ is the consumer price of variety $i$, $Y$ income, and $P \equiv \left( \int_0^n p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$ the price index of manufacturing goods.

The unit factor requirement of the homogeneous good is one unit of labour. This good is freely traded, and since it is chosen as the numeraire

$$p_A = w = 1, \tag{4}$$

$w$ being the nominal wage of workers in all countries.

Shipping the manufactured good involves a frictional trade cost of the "iceberg" form: for one unit of good from country $j$ to arrive in country $k$, $\tau > 1$ units must be shipped. Trade costs are assumed to be equal in both directions. Profit maximization by manufacturing $i$ firms leads to price

$$p_i = \frac{\sigma}{\sigma - 1} a_i, \qquad p_i = \frac{\sigma}{\sigma - 1} \tau a_i \tag{5}$$

in the domestic and foreign market respectively.

Manufacturing firms draw their marginal cost, $a$, from the probability distribution $G(a)$ after having sunk $F_E$ units of labour to develop a new variety.

Having learned their productivity firms decide on entry in the domestic and foreign market. Firms will enter a market as long as the operating profit in this market is large enough to cover the fixed beachhead cost associated with this market. Because of the constant mark-up pricing it is easy to show that operating profits equal sales divided by $\sigma$. Using this and (3), the critical 'cut-off' levels of the marginal costs for the two countries are given by:

$$a_D^{1-\sigma} B = F_D(L), \tag{6}$$

$$a_X^{1-\sigma} \phi B^* = F_X(L^*), \tag{7}$$

$$a_D^{*1-\sigma} B^* = F_D(L^*), \tag{8}$$

$$a_X^{*1-\sigma} \phi B = F_X(L), \tag{9}$$

where $F_D \equiv \delta\sigma\widetilde{F}_D$, $F_X \equiv \delta\sigma\widetilde{F}_X$, $B = \frac{\mu L}{P^{1-\sigma}}$, $B^* = \frac{\mu L^*}{P^{*(1-\sigma)}}$, and $\phi \equiv \tau^{1-\sigma} \in [0, 1]$ represents trade freeness. It is assumed that the fixed market entry cost (beachhead cost) increases in the size of the market $\frac{dF_D}{dL^j}, \frac{dF_X}{dL^j} > 0$. This is a natural assumption, since the marketing costs of establishing a new brand in a large market, such as the US, is much higher than in a small country.

Finally free entry ensures that the ex-ante expected profit of developing a new variety equals the investment cost in both countries:

$$\int_0^{a_D} \left(a_D^{1-\sigma} B - F_D(L)\right) dG(a) + \int_0^{a_X} \left(\phi a_X^{1-\sigma} B^* - F_X(L^*)\right) dG(a) = F_E, \tag{10}$$

$$\int_0^{a_D^*} \left(a_D^{*(1-\sigma)} B^* - F_D(L^*)\right) dG(a) + \int_0^{a_X^*} \left(\phi a_X^{*(1-\sigma)} B - F_X(L)\right) dG(a) = F_E. \tag{11}$$

## 2.2 Solving for the Long-run Equilibrium

We follow Helpman, Melitz and Yeaple (2004) in assuming that the probability density function is Pareto[4]:

$$G(a) = a^k. \tag{12}$$

This implies that the price indices can be written as

$$P^{1-\sigma} = \frac{\beta}{\beta-1}\left(na_D^{1-\sigma} + n^*\phi a_D^{*(1-\sigma)}\left(\frac{a_X^*}{a_D^*}\right)^{k+1-\sigma}\right), \tag{13}$$

$$P^{*(1-\sigma)} = \frac{\beta}{\beta-1}\left(n\phi a_D^{(1-\sigma)}\left(\frac{a_X}{a_D}\right)^{k+1-\sigma} + n^* a_D^{*(1-\sigma)}\right), \tag{14}$$

where $\beta \equiv \frac{k}{\sigma-1} > 1$.

Substituting the cut-off conditions (6), (7), (8), and (9) into the free entry conditions (10) and (11) gives $B$, and $B^*$,

$$B = \left(\frac{F_E F_D^{\beta-1}(L)\cdot(\beta-1)\left(1-\Omega(L^*)\right)}{1-\Omega(L)\Omega(L^*)}\right)^{\frac{1}{\beta}} \tag{15}$$

$$B^* = \left(\frac{F_E F_D^{\beta-1}(L^*)\cdot(\beta-1)\left(1-\Omega(L)\right)}{1-\Omega(L)\Omega(L^*)}\right)^{\frac{1}{\beta}}, \tag{16}$$

where $0 \leq \Omega(L^j) \equiv \phi^\beta\left(\frac{F_X(L^j)}{F_D(L^j)}\right)^{1-\beta} \leq 1$ is a measure of trade freeness.

Using (15), (16) and the cut-off conditions, gives the cut-off marginal costs:

$$a_D^k = \frac{(\beta-1)F_E}{F_D(L)}\left(\frac{(1-\Omega(L^*))}{1-\Omega(L)\Omega(L^*)}\right), \quad a_D^{*k} = \frac{(\beta-1)F_E}{F_D(L^*)}\left(\frac{(1-\Omega(L))}{1-\Omega(L)\Omega(L^*)}\right), \tag{17}$$

$$a_X^k = \frac{(\beta-1)\Omega(L^*)F_E}{F_X(L^*)}\left(\frac{(1-\Omega(L))}{1-\Omega(L)\Omega(L^*)}\right), \quad a_X^{*k} = \frac{(\beta-1)\Omega(L)F_E}{F_X(L)}\left(\frac{(1-\Omega(L^*))}{1-\Omega(L)\Omega(L^*)}\right), \tag{18}$$

From these it is seen that, contrary to the standard model by Melitz (2003), the market size will typically affect the cut-off marginal costs. We will assume that $\frac{F_X^j}{\Omega^j} > F_D^k$ for all $j,k$. As shown below this assumption implies that $a_X < a_D$.

The mass of firms in each country can be calculated using (15), (16), (17), and (18) together with the fact that $B = \frac{\mu L}{P^{1-\sigma}}$, and $B^* = \frac{\mu L^*}{P^{*(1-\sigma)}}$ :

$$n = \frac{\mu(\beta-1)}{F_D(L)\beta}\frac{L\left(1-\Omega(L)\right) - L^*\Omega(L)\left(1-\Omega(L^*)\right)}{\left(1-\Omega(L)\Omega(L^*)\right)\left(1-\Omega(L)\right)} \tag{19}$$

[4]This assumption is consistent with the empirical findings by Axtell (2001).

$$n^* = \frac{\mu(\beta-1)}{F_D(L^*)\beta} \frac{L^*(1-\Omega(L^*)) - L\Omega(L^*)(1-\Omega(L))}{(1-\Omega(L)\Omega(L^*))(1-\Omega(L^*))}. \tag{20}$$

Welfare may be measured by indirect utility, which is proportional to the real wage $\frac{w}{p_A^{1-\mu} P^u}$. Since $p_A = w = 1$ it suffices to examine $P$. Using (13), (17), (18), (19), and (20) we have

$$P = \left(\mu^{-\beta} L^{-\beta} F_D^{\beta-1}(L) F_E(\beta-1) \cdot \frac{1-\Omega(L^*)}{1-\Omega(L)\Omega(L^*)}\right)^{\frac{1}{\beta(\sigma-1)}}. \tag{21}$$

This expression shows that, as in the Melitz (2003) model, home welfare always increases ($P$ decreases) with trade liberalisation; that is with higher $\phi$ or lower $\frac{F_X}{F_D}$.

### 2.2.1 Parametrisation of the beachhead cost

In the following we parametrise the beachhead costs as:

$$\widetilde{F}_D(L^j) = f_D + \left(L^j\right)^\gamma, \quad \widetilde{F}_X(L^j) = f_X + \left(L^j\right)^\gamma, \quad \gamma > 0, \tag{22}$$

This implies that the variable component of the beachhead cost increases in the market size, while the constant term picks up costs that are independent of the market size. It is quite natural that the beachhead cost would have one fixed and one variable component. The constant $f$ could be the fixed cost of standardising a product for a particular market or the cost of producing an advertisement tailored to a particular market with its culture and language. The variable cost term $L^\gamma$ represents that the cost of spreading an advertising message increases with the number of consumers targeted.

# 3 Results

A large number of comparative static results may be derived. We focus here on the more novel aspects of our model, which are related to the effects of market size. From now on the simplified notation $F_D^j \equiv F_D(L^j)$, $F_X^j \equiv F_X(L^j)$, and $\Omega^j \equiv \Omega(L^j)$ is adopted.

## 3.1 Productivity

The first set of results concerns the productivity of exporters and non exporters in the two countries. From (6), and (7)

$$a_D^{\sigma-1} = \frac{B}{F_D}, \quad a_X^{\sigma-1} = \frac{\phi B^*}{F_X^*}. \tag{23}$$

A higher $L^j$ affects the cutoffs via two channels: First it changes the demand facing each firm (via $B$ respective $B^*$), and second, it increases beachhead costs.

From (23)

$$\frac{\partial a_D}{\partial L^*} < 0, \tag{24}$$

since $\frac{\partial B}{\partial L^*} < 0$ by inspection of (15). The intuition being that a larger foreign market implies a larger mass of foreign firms competing in the home market, which decreases the market shares of domestic firms.

The effect of a larger home market on non exporters,

$$\frac{\partial a_D}{\partial L} < 0 \quad \text{for} \quad \phi < 1. \tag{25}$$

The negative signs implies that the higher beachhead cost dominates the effect of a larger home market as shown in appendix 6.4.

Next, from (23)

$$\frac{\partial a_X}{\partial L} < 0, \tag{26}$$

since $\frac{\partial B^*}{\partial L} < 0$. A larger mass of domestic exporters implies stronger competition in the foreign market, and the marginal exporter consequently has to be more productive.

The effect of the foreign market size on the productivity of domestic exporters is, as shown in appendix 6.1, ambiguous:

$$\begin{aligned} \frac{\partial a_X}{\partial L^*} &\leq 0 \quad \text{for} \quad \psi^{\beta-1}\psi^{*(\beta-1)}\left(\beta - (\beta-1)\psi^*\right) \leq \phi^{2\beta} \\ \frac{\partial a_X}{\partial L^*} &> 0 \quad \text{for} \quad \psi^{\beta-1}\psi^{*(\beta-1)}\left(\beta - (\beta-1)\psi^*\right) > \phi^{2\beta}, \end{aligned} \tag{27}$$

where $\psi^j \equiv \frac{F_X^j}{F_D^j}$ measures relative market access (relative beachhead cost) of foreign versus domestic firms. As easily shown, the left hand side of the inequality, determining the sign of the derivative, decreases in $\psi^*$. This means that $a_X$ will always decrease in the foreign market size when the relative beachhead cost in the foreign market is sufficiently high. Referring back to (23), $a_X$ will fall when the effect from higher beachhead cost dominates. For $\psi^*$ close to one, on the contrary, the effect of larger sales dominates, which implies that the marginal exporter can be less productive as the export market increases in size.

*Result 1: The average productivity of exporters as well as non-exporters increase in the size of the domestic market as long as $\phi < 1$. The average productivity of non-exporters also increase in the size of the foreign market. The average productivity of exporters increase in the foreign market size if the beachhead cost of exporters is sufficiently much higher than the beachhead cost of domestic firms in this market.*

The next question is how the relative productivity of firms in the two countries is affected by market size. Note that the productivity of non-exporters in both countries increases as one of the markets grow. As shown in appendix 6.3

$$\left(\frac{a_D}{a_D^*}\right)^k = \frac{F_D^*}{F_D}\left(\frac{1-\Omega^*}{1-\Omega}\right) > 1 \quad for \quad L^* > L, \quad and \quad \Omega^*, \Omega < 1, \tag{28}$$

meaning that domestic producers are more productive in a larger economy. It is also the case that

$$\frac{\partial\left(\frac{a_D}{a_D^*}\right)}{\partial L^*} > 0, \quad for \quad L^* > L, \quad and \quad \Omega^*, \Omega < 1, \tag{29}$$

as shown in appendix 6.2. This implies that the productivity difference between domestic producers in the two economies increases with the difference in market size.

*Result 2: Non-exporters in a large market are on average more productive than non-exporters in a smaller market, and this difference increases with the difference in country size.*

Next using (17), (18) the relative cut-off productivity for non-exporters and exporters in the home country

$$\left(\frac{a_D}{a_X}\right)^k = \frac{F_X^*}{F_D \Omega^*}\left(\frac{1-\Omega^*}{1-\Omega}\right) > 1, \quad for \quad \frac{F_X^j}{\Omega^j} > F_D^k \,\forall\, j,k, \ and \quad \Omega^*, \Omega < 1. \tag{30}$$

There is strong empirical support for exporters being more productive than domestic firms, and we follow Melitz (2003) by making parameters assumptions for this to hold: $\frac{F_X^j}{\Omega^j} > F_D^k$.[5] Also the market size matters here. As shown in appendix 6.5

$$\frac{\partial\left(\frac{a_D}{a_X}\right)}{\partial L} < 0 \quad for \quad \Omega < 1. \tag{31}$$

The larger the home country, the less productive exporters compared to non-exporters. Essentially, the higher fixed cost associated with the larger home market will push up the relative productivity of domestic firms.

*Result 3: Exporters are more productive than producers for the domestic market. However, this effect decreases in the size of the home country.*

#### 3.1.1 Trade costs and exporter productivity

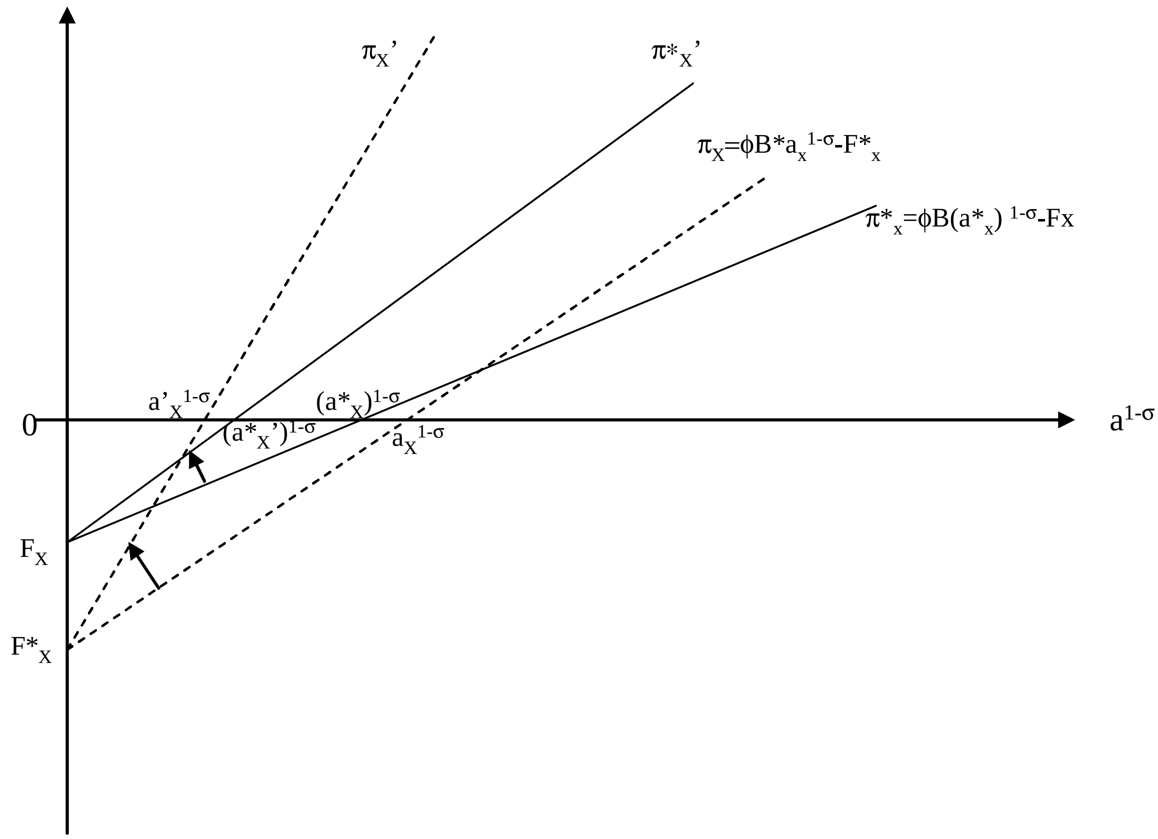
The effect of trade costs on the cut-off productivity in the model is similar to the original Melitz model (see e.g. Baldwin and Forslid 2006): $a_D$ falls and $a_X$ increases with trade freeness. A difference here is that relative market size matters. If the foreign market is larger we have that $a_X < a_D$, even when $\phi = 1$ and $f_X = f_D$.

A question is whether exporters from small or large countries tend to be more productive. As indicated in (27) this may depend on the level of trade costs. From (18), the relative cut-off productivity of exporters from the two countries is given by

$$\left(\frac{a_X}{a_X^*}\right)^k = \frac{F_X \Omega^*}{F_X^* \Omega}\left(\frac{1-\Omega}{1-\Omega^*}\right). \tag{32}$$

It is readily seen that the ratio is one ($a_X = a_X^*$) when the two countries are of equal size. Generally, the value of $f_X$ and $\phi$ determine which country has the lower cut-off productivity in the export sector. Figure 1 illustrates the case when $L^* > L$. The intercept of the profit

[5] The corresponding condition in Melitz (2003) is that $\frac{F_X}{\phi} > F_D$.




lines defines the cut-off $a_X$ and $a_X^*$. From (7) and (9) the relative slope of the two countries' exporters profit lines is

$$\frac{\phi B}{\phi B^*} = \left( \frac{F_X^{*\beta-1}\Omega^*(1-\Omega)}{F_X^{\beta-1}\Omega(1-\Omega^*)} \right)^{\frac{1}{\beta}} > 1. \tag{33}$$

From (33) it is easy to show that the relative slope increases in $\phi$, meaning that the $\pi^*$ curve in Figure 1 rotates more than the $\pi$ curve (in the counter-clockwise direction) as trade costs fall. This implies that $\frac{a_X}{a_X^*}$ decreases in $\phi$ :[6]

$$\frac{\partial \left( \frac{a_X}{a_X^*} \right)}{\partial \phi} < 0. \tag{34}$$

Figure 3.1.1 displays an increase in $\phi$ large enough to make initially more productive domestic exporters less productive than foreign exporters. It is easy to show (see appendix 6.7) that $a_X < a_X^*$ for $\phi = 0$, and that $a_X \geqslant a_X^*$ for $\phi = 1$, which implies that the shift in productivity ranking between the two countries, shown in Figure 3.1.1, will occur for a large enough change in $\phi$.

The intuition for (34) is that exporters from the large country faces a lower beachhead cost, but has a smaller foreign market. The fixed cost is relatively more important than the market

[6] A formal proof is found in appendix 6.6.

size when trade costs are high, which implies that the cut-off marginal cost for exporters from the large country tends to be higher ($a_X < a_X^*$) in this case. When trade costs are low market size becomes relatively more important, to the benefit of exporters from the small country. The cut-off marginal cost for these exporters therefore becomes lower than for exporters from the large country ($a_X > a_X^*$).

Essentially the same logic applies to a change in $f_X$. In Figure 3.1.1 a higher $f_X$ implies that all profit lines shift down by an equal amount, which implies that all intercepts move to the right. However, the intercept of a more shallow (less steep) line must move more than that of a steeper line. The foreign profit line is less steep than the domestic and consequently we have that

$$\frac{\partial\left(\frac{a_X}{a_X^*}\right)}{\partial f_X} > 0. \tag{35}$$

*Result 4: The relative productivity of exporters in a small country compared to exporters in a large country decreases as trade costs fall. The same holds for a fall in the fixed beachhead cost ($f_X$) of exporters. Exporters from a small country are more productive close to autarky and less productive at free trade compared to exporters from a large country.*

### 3.2 Trade volume

The next result concerns the relationship between country size and manufacturing export share. A home exporting firm with marginal cost $a$, sells $a^{1-\sigma}\phi B^*$ in the foreign market. Using (7), the total export volume from home is

$$V_X = \int_0^{a_X} a^{1-\sigma} dG(a \mid a_D) \cdot \frac{F_X^*}{a_X^{1-\sigma}} = \left(\frac{a_X}{a_D}\right)^k \frac{\beta}{\beta-1} F_X^* n. \tag{36}$$

Similarly the total production volume for the home market is

$$V_D = \int_0^{a_D} a^{1-\sigma} dG(a \mid a_D) \cdot \frac{F_D}{a_D^{1-\sigma}} = \frac{\beta}{\beta-1} F_D n. \tag{37}$$

The export share

$$S_X = \frac{V_X}{V_X + V_D} = \frac{1}{1 + \frac{B}{B^*}\frac{1}{\phi}\left(\frac{a_D}{a_X}\right)^{k\frac{\beta}{\beta-1}}}. \tag{38}$$

The export share increases in the relative size of the foreign market ($\frac{B^*}{B}$), in trade freeness, and in the ratio of exporting firms to domestic producers $\left(\frac{a_X}{a_D}\right)^k$. The export share may also be written more compactly as,

$$S_X = \frac{\Omega^*(1-\Omega)}{1-\Omega^*\Omega}. \tag{39}$$

Differentiating with respect to country size gives

$$\frac{\partial S_X}{\partial L} = \frac{\Omega^* (\Omega^* - 1)}{(1 - \Omega^* \Omega)^2} \frac{\partial \Omega}{\partial L} < 0, \tag{40}$$

$$\frac{\partial S_X}{\partial L^*} = \frac{1 - \Omega}{(1 - \Omega^* \Omega)^2} \frac{\partial \Omega^*}{\partial L^*} > 0. \tag{41}$$

*Result 5: The manufacturing export share of a country decreases in its own size, and increases in the trade partners size.*

This result seems very natural, and at least at an aggregate level it is generally taken for granted. By setting $\gamma = 0$ in our model market size effects are neutralised and we have the original Melitz (2003) model. From (39) $\gamma = 0$ implies that $S_X = \frac{\Omega}{1+\Omega}$, where $\Omega = \phi^\beta \left( \frac{f_X}{f_D} \right)^{1-\beta}$. Thus, the export share is independent of the country size in the original model.

Next, note that for $f_X = f_D$, $\Omega^* = \Omega = 1$. This means, from (39), that $S_X = S_X^*$; that is, manufacturing export shares converge as $f_X$ approaches $f_D$. The intuition for this follows from the effects visible in (38). First, the fixed component of the beachhead cost $f_X$ is more important in a small market. A falling $f_X$ therefore increases market access relatively more in a small market, which means that $\frac{B}{B^*}$ approaches one. Second, a fall in $f_X$, makes export easier while increasing import competition in both countries. This results in a decreasing $a_D^j$, and an increasing $a_X^j$, and as a result $\frac{a_D^j}{a_X^j}$ approaches one in both countries.

*Result 6:Falling relative beachhead costs ($f_X$ converging to $f_D$) implies converging manufacturing export shares.*

## 4 Empirical Analysis

In this section we empirically test aspects of the theoretical model of the preceeding analysis. In particular we focus on testing the new predictions of our model related to the effects of market size. Ideally the predictions of our model should be tested in a cross-country firm level data set. This is type of data is, however, not yet available. We use cross-section data rather than firm level data for an individual country, since the novel aspect of our results are related to country or market size differences.

We use two datasets: the OECD STAN Industrial Database and the World Bank's World Development Indicators (WDI). The STAN database has rich sectoral data consisting of 28 manufacturing sectors but fewer countries, whereas the WDI contains considerably more countries but only contains information at the country level.

We use data from the STAN for industry variables such as output, value added, total employment and trade. For population measures when using STAN we use the OECD population data. Theoretically, the dataset stretches from 1970 to 2003 but we limit our analysis to the period from 1980 to 2001 due to many missing values outside these years.[7] The WDI data covers

[7] Any variable given in current prices in local currencies is converted into US dollar equivalents by using annual

all countries over the period from 1960 to 2005 and gives information about trade, output and population.

### 4.1 Average productivity

The average productivity of non-exporters as well as exporters increases in the home market size, as shown above in Result 1. The average overall productivity in the model may be expressed as[8]:

$$\overline{\varphi} = \left( s_D \int_0^{a_D} a^{1-\sigma} dG(a \,|a_D\,) + s_X \int_0^{a_X} a^{1-\sigma} dG(a \,|a_D\,) \right)^{\frac{1}{\sigma-1}}, \tag{42}$$

where $s_D$ is the share of home producers that sell domestically only and and $s_X$ the share that export. Since the ratio of exporters to non-exporters is $\left(\frac{a_X}{a_D}\right)^k$, $s_D = \frac{1}{1+\left(\frac{a_X}{a_D}\right)^k}$, and $s_X = \frac{\left(\frac{a_X}{a_D}\right)^k}{1+\left(\frac{a_X}{a_D}\right)^k}$, we can rewrite (42) as:

$$\overline{\varphi} = \frac{1}{a_D} \left( \frac{k}{k-\sigma+1} \right)^{\frac{1}{\sigma-1}} \left( \frac{1+\left(\frac{a_X}{a_D}\right)^{2k+1-\sigma}}{1+\left(\frac{a_X}{a_D}\right)^k} \right)^{\frac{1}{\sigma-1}}. \tag{43}$$

From (43) it is seen that average productivity increases in $L$ since from (**??**) $\frac{\partial a_D}{\partial L} < 0$, from (31) $\frac{\partial \left(\frac{a_X}{a_D}\right)}{\partial L} > 0$ , and $k - \sigma + 1 > 0$.

To test this prediction, we run a regression of the following specification:

$$\log \widetilde{\varphi}_{ist} = \beta_0 + \beta_L \log L_{it} + \beta_K \log K_{ist} + \beta_D \log D + \varepsilon_{ist}. \tag{44}$$

Here, $\widetilde{\varphi}_{ist}$ denotes aggregate labour productivity in country $i$ in sector $s$ and year $t$. $L_{it}$ is the national population size of country $i$ in year $t$. $K_{ist}$ is the amount of capital used and $D$ is a set of dummies, which ones specifically follows.

We control for sectors by using the set $D_s$ in all regressions, since $f_D$, $f_X$, and $\gamma$ are expected to vary among sectors. We also use specifications with country dummies to control for the possibility that population growth within countries affect aggregate productivity differently than cross sectional differences in population.

We use two measures of labour productivity: (1) output divided by employment and (2) value added divided by employment.[9] A problem is that employment is reported by different countries in different (but similar) ways. We will use the standard measure which covers most countries which is called total employment in the database.

average exchange rates from the International Financial Statistics Database at the IMF.

[8] See Melitz (2003).

[9] Pavcnik (2002) use the semiparametric method from Olley & Pakes (1996) to estimate productivity. However, we do not have firm level data which would be required for this method

We use population as measure of country size when estimating the effect of country size on productivity. This is, first, because population can be considered an exogenous variable for our purposes and, second, it is consistent with the treatment of country size in our model. Were we to use, for example, GDP instead this would depend both on population size and aggregate productivity.

The error term, $\varepsilon_{ist}$, is assumed independent between combinations of countries and sectors, but this assumption is relaxed in a specific country. We therefore cluster on country and year pairs. This is since the highest aggregation level is at the country level when we use population. The number of clusters is therefore as many as there are combinations of countries and years.

The STAN database reports output and value added in two ways. The first is in values at current prices and in the local currencies and the second is as a volume index typically based at 100 in 1995. Since the volume sample is significantly smaller than with values both measures are used in the estimation. When using values there is therefore a clear risk that price levels endogenously affect the estimates. We run these pooled OLS regressions[10], as in (44) with and without controlling for capital, for the sample 1980 to 2001 with a set of dummies for all combinations of sectors and years. In this way global sectoral shocks are captured. We also run the regression year by year (without capital) and plot the result in a graph. When the volume index is used we cannot use the same regression setup. Instead a set of dummies specific for each pair of country and sector is used. In this way the denominator (output in values divided by the price level at the year of rebasing) is captured (since the left hand side variable, productivity, is in logarithms) and we only look at the changes.

| | $\frac{\text{Output}}{\text{Worker}}$ Values (1) | $\frac{\text{Output}}{\text{Worker}}$ Volumes (2) | $\frac{\text{Output}}{\text{Worker}}$ Values (3) | $\frac{\text{Output}}{\text{Worker}}$ Volumes (4) | $\frac{\text{V.A.}}{\text{Worker}}$ Values (5) | $\frac{\text{V.A.}}{\text{Worker}}$ Volumes (6) | $\frac{\text{V.A.}}{\text{Worker}}$ Values (7) | $\frac{\text{V.A.}}{\text{Worker}}$ Volumes (8) |
|---|---|---|---|---|---|---|---|---|
| Population | 0.299*** | 7.294*** | −0.011 | 5.812*** | 4.84*** | 2.419*** | 0.891*** | 0.655** |
| | (0.094) | (0.366) | (0.008) | (0.425) | (0.369) | (0.228) | (0.283) | (0.272) |
| Capital | | | 0.976*** | 0.192*** | | | 0.658*** | 0.328*** |
| | | | (0.004) | (0.027) | | | (0.034) | (0.027) |
| Dummies | | | | | | | | |
| Sector & year | Yes | No | Yes | No | Yes | No | Yes | No |
| Country & sector | No | Yes | No | Yes | No | Yes | No | Yes |
| # obs | 8931 | 3564 | 3381 | 2566 | 9050 | 5622 | 3379 | 3331 |
| R squared | 0.09 | 0.98 | 0.98 | 0.99 | 0.98 | 0.97 | 0.99 | 0.98 |

Note: Robust standard errors in parentheses. Errors are clustered on country and year pairs. *

[10] Local currencies are converted into US dollars using exchange rate data from the IMF's International Financial Statistics (IFS) database.

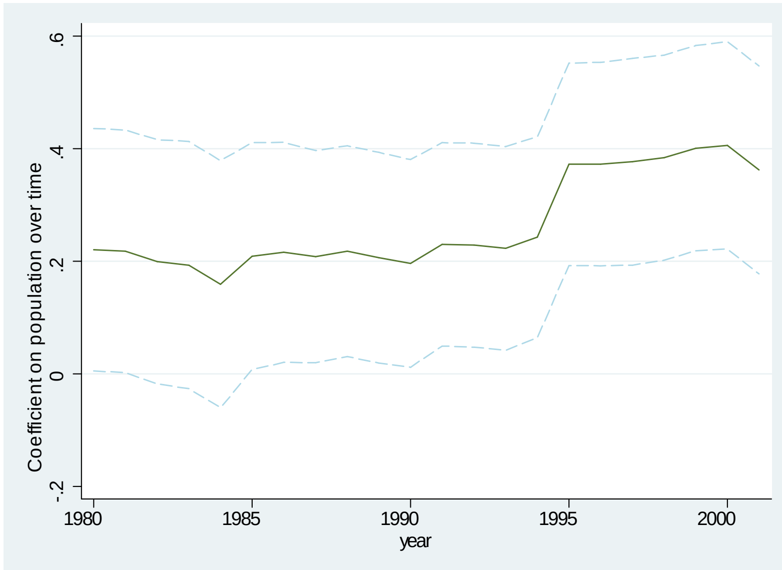



Figure 1: The coefficient on population increases over time. Source: OECD.

significant at 10%; ** significant at 5%; *** significant at 1%. Included years for OECD are 1980 to 2002.

The hypothesis that productivity is positively associated with country size is supported both for the cross sectional analysis with values in (1) and (5). When only internal population growth in countries is considered but when we can use volume data instead, as in (2) and (6), we get an equally strong coefficient on population size. The effect is upheld also when capital is controlled for as in (4), (7) and (8) but not in (3). However, the estimate in (7) is more important than in (3) since it uses value added in production as a dependent variable which is closer to our notion oif productivity in the model. There are two problems, however, when we include capital. First, the sample size shrinks by around two thirds since most countries do not report this variable, and the remaining sample is much less evenly spread across countries. Second, there is an obvious causality problem; if productivity is high in a sector this would attract capital. Or, as in Markusen (1983), exporting sectors can sometimes attract factors of production, capital here, against Heckscher Ohlin predictions. Because of such endogeneity problems the estimates where capital is included should be treated with some caution.

We also run (44) on a year by year basis as shown in the following figure (with 95% confidence intervals) with sectoral dummies. Figure 1 shows that the coefficient on population increases with time but also that it is positive significant throughout the period. The increase should also be treated with caution since it may simply reflect inflation in nominal values of value added which cannot be controlled for without sector specific price indices.

### 4.2 Country size and trade shares of GDP

As shown above manufacturing export shares are decreasing in the home market size (Result 4). This implies that large countries have below average export shares and small countries above average export shares.

One cited effect of globalisation is that it makes the world more homogeneous in many aspects. This should in principle make it easier to enter foreign markets, meaning that $f_X$ over time would converge towards $f_D$. Except for different language and culture, the difference between $f_X$ and $f_D$ may reflect the cost of adopting a product to foreign technical standards. At least within the EU, which constitutes a substantial part of OECD, the removal of non-tariff barriers, or technical barriers to trade, has been a major objective. By now common standards on almost all types of manufacturing products have been adopted. A convergence between $f_X$ and $f_D$, implies from Result 6, that manufacturing export shares should converge over time. Below, we test the predictions in several steps as well as the notion that trade to GDP ratios decrease in country size.

#### 4.2.1 Larger countries have lower export to GDP ratios

It is somewhat of a stylised fact that larger countries on average have lower ratios of trade flows relative to their GDP, as predicted by the model. However, this is easy to test in our data. We do so by running the simple regression

$$\log\left(\frac{X}{Y}\right)_{ist} = \beta_0 + \beta_1 L_{it} + \varepsilon_{ist}. \tag{45}$$

The regression is first run at sectoral level by using the STAN database. Column 1 shows the regression of trade shares over GDP on a sectorial level in the year of 2001. The regression includes fixed effects for sectors. We also test the hypothesis on country level data by using the World Bank's World Development Indicators for the same year. By doing this many more countries can be included, although the regression is then on the country level and not sectorial. The result from the regression using the WDI data is found in column 2. The coefficients for population are highly significant and of the expected sign.

| | $\frac{X}{Y}$ (1) | $\frac{X}{Y}$ (2) |
|---|---|---|
| Datasets | STAN | WDI |
| $POP$ | $-0.16^{***}$ | $-0.10^{***}$ |
| | (0.025) | (0.035) |
| Sector dummies | Yes | NA |
| # obs | 605 | 158 |
| R squared | 0.38 | 0.24 |

Note: Standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%.

#### 4.2.2 Export to GDP ratios converge over time

Our model also predicts that export to GDP ratios will converge over time given that entry (beachhead) costs into foreign markets are falling over time. We again test this assumption using our two datasets.

The first approach is to regress the annual change (first difference) in the export to GDP ratio in a specific sector or country on a dummy that takes the value 1 if that sector or country has a lower export share than the average interacted with a time variable:

$$\Delta s_{ist} = \beta_0 + \beta_1 L_{it} + \beta_2 D_{ist} + \beta_3 * t_t + \beta_4 * D_{ist} * t_t + \varepsilon_{ist}$$

where $s_{ist} = \log\left(\frac{X+M}{Y}_{ist}\right)$ and $\Delta s_{ist} = s_{ist} - s_{ist-1}$. The time variable $t_t$ takes value 1 if it is the first year in the sample, 2 if the second etc. If our hypothesis were correct we would (with fixed effects for all sector and year combinations) find a positive value for $\beta_4$. The result is reported in columns 1 for the OECD data and 2 for the WDI data. The coefficient on the interacted variable is significantly positive in both columns as predicted.

| | $\Delta s_{ist}$ | $\Delta s_{ist}$ |
|---|---|---|
| Datasets | OECD | WDI |
| $POP_{ist}$ | $-0.081^{***}$ | $-0.006^{***}$ |
| | (0.013) | (0.004) |
| $D_{ist}$ | $-1.744^{***}$ | $-1.05^{***}$ |
| $t$ | 0.03 | $0.011^{***}$ |
| $D_{ist} * t$ | $0.022^{***}$ | $0.002^{***}$ |
| | (0.005) | (0.001) |
| Sector dummies | Yes | NA |
| # obs | 11328 | 5800 |
| R squared | 0.7 | 0.6 |

Note: Standard errors in parentheses. Errors are clustered on country and year pairs for the OECD. * significant at 10%; ** significant at 5%; *** significant at 1%. Included years for OECD are 1980 to 2002 and for WDI 1960 to 2005.

Our second approach is to regress the first difference in export shares on its own lagged value in levels:

$$\Delta s_{ist} = \beta_0 + \beta_1 s_{ist-1} + \beta_2 \Delta L_{it} + \beta_3 D_s + \varepsilon_{ist}$$

with fixed effects for sectors when using the OECD data. We also cluster on country sector pairs in the OECD data and on countries in the WDI data. Our model would predict a negative value of $\beta_1$ for convergence and a negative value of $\beta_2$ if export shares are lower in larger countries. It could be, however, that the errors are serially correlated and therefore we include lags up to the degree of $p = 5$:

$$\Delta s_{ist} = \beta_0 + \sum_{i=1}^{p} \beta_{1i} s_{ist-i} + \beta_2 \Delta L_{it} + \beta_3 D_s + \varepsilon_{ist}.$$

The results are shown in the following figure. The sign on the first lag of the export share is negative and significant suggesting convergence. Especially, the result is upheld also in the regressions with five lags suggesting that serial correlation is not of a worryingly high magnitude.

| | $\Delta s_{ist}$ | $\Delta s_{ist}$ | $\Delta s_{ist}$ | $\Delta s_{ist}$ |
|---|---|---|---|---|
| Datasets | OECD | WDI | WDI | WDI |
| $s_{ist-1}$ | $-0.101^{***}$ | $-0.04^{***}$ | $-0.04^{***}$ | $-0.093^{***}$ |
| | (0.034) | (0.006) | (0.004) | (0.028) |
| $\Delta POP_{ist}$ | 0.99 | | $-0.036$ | $-0.157$ |
| | (1.026) | | (0.026) | (0.304) |
| $s_{ist-2}$ | 0.019 | | | $-0.068^{*}$ |
| $s_{ist-3}$ | 0.059$*$ | | | $0.101^{***}$ |
| $s_{ist-4}$ | $-0.041$ | | | $-0.002$ |
| $s_{ist-5}$ | $-0.010$ | | | 0.031 |
| Sector dummies | Yes | NA | NA | NA |
| # obs | 8072 | 5614 | 5614 | 5614 |
| R squared | 0.04 | 0.01 | 0.01 | 0.01 |

Note: Standard errors in parentheses. Errors are clustered on country and year pairs. * significant at 10%; ** significant at 5%; *** significant at 1%. Included years for OECD are 1980 to 2002 and for WDI 1960 to 2005.

Our third approach is to use the initial value for which we have data and regress the first differences in export shares on that historical level

$$\Delta s_{ist} = \beta_0 + s_{is0} + \beta_2 \Delta L_{it} + \beta_3 D_s + \varepsilon_{ist}.$$

In the following figure it is seen that the growth rate of export shares depend negatively on the initial level in 1960, suggesting convergence both within the OECD at the sectoral level and on a global scale in the WDI data.

| | $\Delta s_{ist}$ | $\Delta s_{ist}$ | $\Delta s_{ist}$ | $\Delta s_{ist}$ |
|---|---|---|---|---|
| Datasets | OECD | OECD | WDI | WDI |
| $s_{i,1960}$ | $-0.015^{***}$ | $-0.016^{***}$ | $-0.01^{***}$ | $-0.01^{***}$ |
| | (0.003) | (0.003) | (0.004) | (0.004) |
| $\Delta POP_{it}$ | | $-0.653^{**}$ | | $-0.047$ |
| | | (0.579) | | (0.199) |
| Sector dummies | Yes | Yes | NA | NA |
| # obs | 8371 | 8371 | 3625 | 3625 |
| R squared | 0.01 | 0.01 | 0.0 | 0.0 |

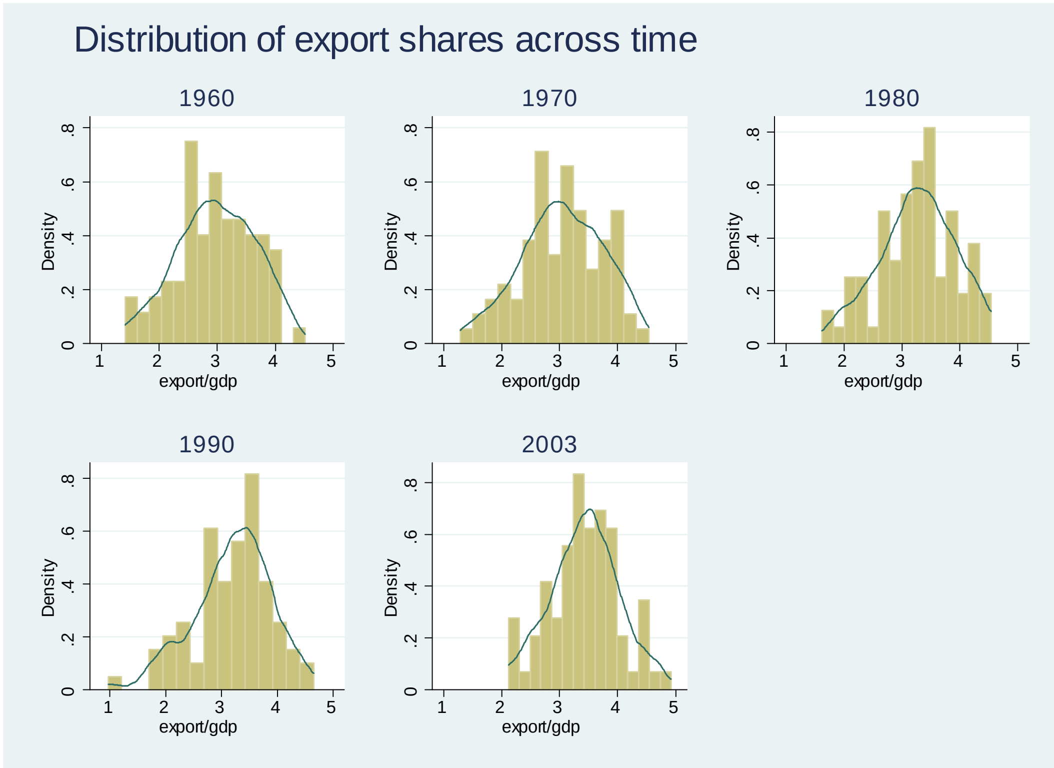



Figure 2: The distribution of logged values of $\frac{X+M}{Y} * 100$ becomes more narrow as time progresses. Source: World Development Indicators.

Note: Standard errors in parentheses. Errors are clustered on country and year pairs. * significant at 10%; ** significant at 5%; *** significant at 1%. Included years for OECD are 1980 to 2002 and for WDI 1960 to 2005.

Finally the effect is visible by graphically examining the shift in the distribution of export shares globally. We restrict the WDI sample to only include countries for which there is data from 1960 until 2003 and construct five histograms displaying the distribution of export shares globally in the years 1960, 1970, 1980, 1990 and 2003. We also include a kernel density estimate in the graphs. The result is presented in figure 2. It can be seen that the distribution becomes more narrow as time progresses and trade costs fall.

## 4.3 Country size and export premia

Result 1 in the theoretical section states that domestic firms in a large market are on average more productive than domestic firms in a smaller market, and that this difference increases with the difference in country size. A proper test of this result would require cross-country firm level data, which is not yet available. There are however a number of productivity studies on the country level using firm level data. Schank, Schnabel and Wagner (wp 2006) offers a literature overview where they measure the productivity premium awarded to exporters compared to non

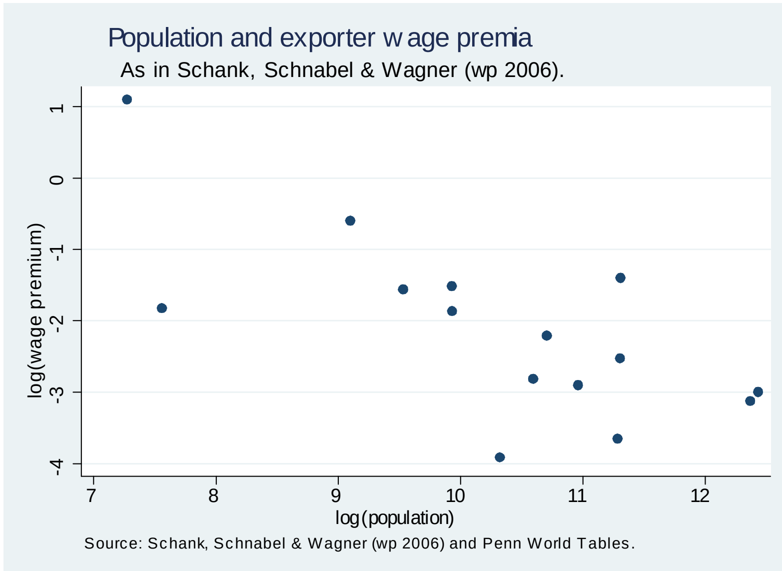



Figure 3: Export premia decrease in country size.

exporters. Typically, a regression is run on firm level data with the following properties:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 D_{it} + \varepsilon_{it}, \tag{46}$$

where $Y_{it}$ is some measure of productivity, $X_{it}$ is a set of control variables and $D_{it}$ is a dummy which takes the value 1 if the firm is an exporter and 0 otherwise. The size of $\beta_2$ is therefore the measure of the productivity premium. Figure 3 plots the size of $\beta_2$ versus population size in countries, where we have added an observation for Sweden using the regression above with firm level data provided by Statistics Sweden. Of course, it must be acknowledged that all regressions here are not done with exactly the same methodology or fully comparable data so using the study here rather provides guidance rather than rigid evidence. Nevertheless figure 3 shows a negative correlation between the export premium and population size. Running a regression on these data returns a beta estimate of size $-0.605$ with a t value of $-3.68$ which is significant.

## 5 Conclusion

This paper has explicitly modeled a market size dependent beachhead cost in the heterogenous firms and trade model by Melitz (2003). Interpreting the beachhead cost as the advertising cost of establishing a new brand in an market, it is natural to assume that the cost is increasing in the size of the market, since it is more costly to reach a higher number of consumers.

The introduction of market size dependent beachhead costs leads to a number of new results. The productivity of domestic firms as well as of exporters will depend on market size, and so will export shares. In particular, we show that domestic firms in a large market are more productive than in a smaller market. Second, as in the standard model, exporters are more productive than producers for the domestic market, but this effect decreases in the size of the home country. Third, compared to exporters from a large country, exporters from a small country are more productive close to autarky and less productive at free trade. Finally, we show that the export share of a country decreases in its own size, and increases in the trade partners size. This last effect decreases as trade costs fall, indicating that we would expect converging trade shares over time, given that trade costs fall over time.

In the empirical section of the paper we test a number of predictions of our model. In particular we here focus on results related to country size, which are new compared to the standard model. This implies that we need to use cross-country data. First we show how average productivity is positively correlated with country size, in accordance with our model. Second, a number of tests support for the model generated hypothesis that, under the assumption that trade costs are decreasing over time, trade shares should converge over time. Finally, our model predicts that the export premia should decrease in country size. Using a survey of country level productivity estimates using firm level data we find a negative correlation between population and export premia. Taken together our empirical tests provide a strong case for our model specification.

# 6 Appendix

## 6.1 $\frac{\partial a_X}{\partial L^*}$

From (18)

$$a_X^k = \frac{(\beta-1)\,\Omega^* F_E}{F_X^*}\left(\frac{(1-\Omega)}{1-\Omega\Omega^*}\right) = (\beta-1)\,F_E(1-\Omega)\frac{1}{F_X^*(\frac{1}{\Omega^*}-\Omega)} \tag{47}$$

The sign of $\frac{\partial a_X}{\partial L^*}$ is therefore determined by the sign of

$$\frac{\partial}{\partial L^*}[F_X^*(\frac{1}{\Omega^*}-\Omega)] \tag{48}$$

$$= \frac{\partial}{\partial L^*}[F_X^{*\beta}F_D^{*1-\beta}\phi^{-\beta} - F_X^*\Omega]. \tag{49}$$

Now

$$\begin{aligned}
\frac{\partial}{\partial L^*}[F_X^{*\beta}F_D^{*1-\beta}\phi^{-\beta} - F_X^*\Omega] &\lesseqgtr 0 \\
&\Leftrightarrow \\
\left(\frac{F_X^*}{F_D^*}\right)^\beta\left(\beta\frac{F_D^*}{F_X^*} - (\beta-1)\right) &\lesseqgtr \Omega\phi^\beta \\
&\Leftrightarrow \\
\beta - \frac{F_X^*}{F_D^*}(\beta-1) &\lesseqgtr \Omega\Omega^*.
\end{aligned}$$

## 6.2 $\frac{\partial}{\partial L^*}\left(\frac{a_D}{a_D^*}\right)^k > 0$ for $\Omega^*, \Omega < 1$

Proof:

From (28)

$$\left(\frac{a_D}{a_D^*}\right)^k = \frac{F_D^*}{F_D}\left(\frac{1-\Omega^*}{1-\Omega}\right).$$

Differentiating w.r.t. $L^*$ gives:

$$\frac{\partial}{\partial L^*}\left(\frac{F_D^*}{F_D}\left(\frac{1-\Omega^*}{1-\Omega}\right)\right) = \frac{\gamma L^{*\gamma-1}}{F_D\,(1-\Omega)}\left(1-\Omega^*-(\beta-1)\,\Omega^*\left(1-\Omega^{*\frac{1}{\beta-1}}\phi^{\frac{\beta}{1-\beta}}\right)\right). \tag{50}$$

The sign of the derivative depends on the sign of the term:

$$\left(1-\Omega^*-(\beta-1)\,\Omega^*\left(1-\Omega^{*\frac{1}{\beta-1}}\phi^{\frac{\beta}{1-\beta}}\right)\right). \tag{51}$$

The first and second order conditions for a minimum of this term w.r.t. $\Omega\,(L^*)$ are:

$$\begin{aligned}
\frac{\partial}{\partial\Omega^*}\left(\Omega^*\left(1+(\beta-1)\left(1-\Omega^{*\frac{1}{\beta-1}}\phi^{\frac{\beta}{1-\beta}}\right)\right)\right) &= \beta\left(\Omega^{*\frac{1}{\beta-1}}\phi^{\frac{\beta}{1-\beta}}-1\right)=0 \\
\frac{\partial^2}{\partial^2\Omega^*}\left(\Omega^*\left(1+(\beta-1)\left(1-\Omega^{*\frac{1}{\beta-1}}\phi^{\frac{\beta}{1-\beta}}\right)\right)\right) &= \frac{\beta}{\beta-1}\Omega^{*\frac{1}{\beta-1}-1}\phi^{\frac{\beta}{1-\beta}} > 0.
\end{aligned} \tag{52}$$

The minimum is, thus, given by $\Omega^* = 1$ (since $\Omega^* = 1 \Longleftrightarrow \phi = 1$). Substituting $\Omega^* = 1$ into (50) gives $\frac{\partial}{\partial L^*}\left(\frac{a_D}{a_D^*}\right)^k = 0$. Consequently it must be that $\frac{\partial}{\partial L^*}\left(\frac{a_D}{a_D^*}\right)^k > 0$ for $\Omega^*, \Omega < 1$.

### 6.3 $\left(\frac{a_D}{a_D^*}\right)^k > 1$ iff $L^* > L$ for $\Omega^*, \Omega < 1$

Proof:

First

$$L^* = L \Longleftrightarrow \left(\frac{a_D}{a_D^*}\right)^k = \frac{F_D^*}{F_D}\left(\frac{1-\Omega^*}{1-\Omega}\right) = 1.$$

That $L^* = L \Longleftrightarrow \left(\frac{a_D}{a_D^*}\right)^k > 1$ for $\Omega^*, \Omega < 1$ now follows from $\frac{\partial}{\partial L^*}\left(\frac{a_D}{a_D^*}\right)^k > 0$ for $\Omega^*, \Omega < 1$.

### 6.4 $\frac{\partial a_D^j}{\partial L^j} < 0$

From (29) we have that

$$\frac{\partial\left(\frac{a_D}{a_D^*}\right)}{\partial L^*} = \frac{\partial a_D}{\partial L^*}\frac{1}{a_D^*} - \frac{a_D}{\left(a_D^*\right)^2}\frac{\partial a_D^*}{\partial L^*} > 0. \tag{53}$$

Since from (17) $\frac{\partial a_D}{\partial L^*} < 0$, (53) holds iff $\frac{\partial a_D^*}{\partial L^*} < 0$

### 6.5 $\frac{\partial\left(\frac{a_D}{a_X}\right)}{\partial L} < 0 \quad for \quad \Omega < 1$.

Proof:

$$\frac{\partial}{\partial L}\left(\frac{a_D}{a_X}\right)^k = \gamma L^{\gamma-1}\frac{F_X^*}{F_D^2}\frac{(1-\Omega^*)}{\Omega^*(1-\Omega)}\left((\beta-1)\frac{\Omega\left(1-\frac{F_D}{F_X}\right)}{(1-\Omega)} - 1\right). \tag{54}$$

The sign of (54) will depend on the sign of the term:

$$\Theta \equiv \left(\frac{(\beta-1)\,\Omega\left(1-\frac{F_D}{F_X}\right)}{(1-\Omega)} - 1\right) \tag{55}$$

The F.O.C. when maximising $\Theta$ w.r.t. $\phi$ is:

$$\frac{(\beta-1)\,\beta\Omega\left(1-\frac{F_D}{F_X}\right)}{\phi\,(1-\Omega)} - \frac{(\beta-1)\,\beta\Omega^2\left(1-\frac{F_D}{F_X}\right)}{\phi\,(1-\Omega)^2} = 0 \Longleftrightarrow 1 - \frac{\Omega}{(1-\Omega)} = 0 \tag{56}$$

So the only stationary point is $\Omega = 1$. Furthermore $\Theta(\Omega = 0) = -1$ and $\lim_{\Omega(L)\to 1}\Theta = 0$. It therefore follows that for $\Omega \in [0, 1)$:

$$\frac{d}{dL}\left(\frac{a_D}{a_X}\right)^k < 0.$$

## 6.6 $\frac{a_X}{a_X^*}$ increases (decreases) in $\phi$ if $L^* > L$ $(L^* < L)$.

Proof:

$$\begin{aligned}
\frac{\partial}{\partial\phi}\left(\frac{a_X}{a_X^*}\right)^k &= \frac{d}{d\phi}\left(\frac{F_X\Omega^*(1-\Omega)}{F_X^*\Omega(1-\Omega^*)}\right) \\
&= \beta\phi^{-1}\frac{F_X}{F_X^*}\frac{\Omega^*}{(1-\Omega^*)}\left(\frac{\Omega^*}{1-\Omega^*}\frac{1-\Omega}{\Omega}-1\right) \\
&= \begin{cases} >0 & L^*>L \\ <0 & L^*<L \end{cases}.
\end{aligned}$$

■

## 6.7 $a_X < a_X^*$ for $\phi = 0$, and $a_X > a_X^*$ for $\phi = 1$ if $L^* > L$.

First from (32)

$$\left.\left(\frac{a_X}{a_X^*}\right)^k\right|_{\phi=0} = \frac{F_X}{F_X^*} < 1 \quad for \quad L^* > L \tag{57}$$

Second

$$\frac{\partial}{\partial L^*}\left(\frac{a_X}{a_X^*}\right)^k = \gamma L^{*\gamma-1}\frac{F_X\Omega^*(1-\Omega)}{F_X^{*2}\Omega(1-\Omega^*)}\left(-1+(1-\beta)\left(1-\Omega^{*-\frac{1}{\beta-1}}\right)\left[1+\frac{\Omega^*}{1-\Omega^*}\right]\right) \tag{58}$$

The sign of the derivative depends on the sign of the term inside the bracket in (58), which for $\phi = 1$, may be written as:

$$(\beta-1)(\psi^*-1)\frac{1}{1-(\psi^*)^{1-\beta}}-1, \quad \psi^* \equiv \frac{F_X^*}{F_D^*} \in [1,\infty) \tag{59}$$

(59) has no stationary point when $\psi^* > 1$, and is positive for large $\psi^*$. Moreover

$$\lim_{\psi^*\to1}(\beta-1)(\psi^*-1)\frac{1}{1-(\psi^*)^{1-\beta}}-1 = 0 \tag{60}$$

We therefore have that

$$\left.\left(\frac{a_X}{a_X^*}\right)^k\right|_{\phi=1} \geqslant 1 \quad for \quad L^* > L. \tag{61}$$

■

### 6.8 $\frac{a_X}{a_X^*}$ increases (decreases) in $f_X$ if $L^* > L$ ($L^* < L$)?

Proof:

$$\begin{aligned}
\frac{d}{df_X}\left(\frac{a_X}{a_X^*}\right)^k &= \frac{\Omega^*(1-\Omega)}{(f_X+L^{*\gamma})\,\Omega\,(1-\Omega^*)}\left(-\beta\frac{(f_X+L^\gamma)}{(f_X+L^{*\gamma})}+\beta+(1-\beta)\frac{\Omega^*}{(1-\Omega^*)}\frac{(f_X+L^\gamma)}{(f_X+L^{*\gamma})}-(1-\beta)\frac{\Omega}{(1-\Omega)}\right)\\
&= \frac{\Omega^*(1-\Omega)}{(f_X+L^{*\gamma})\,\Omega\,(1-\Omega^*)}\left(\beta\left(1-\frac{(f_X+L^\gamma)}{(f_X+L^{*\gamma})}\right)+(\beta-1)\left(\frac{\Omega}{(1-\Omega)}-\frac{\Omega^*}{(1-\Omega^*)}\frac{(f_X+L^\gamma)}{(f_X+L^{*\gamma})}\right)\right)
\end{aligned}$$

$$\beta\left(L^{*\gamma}-L^\gamma\right)+(\beta-1)\left(\frac{\Omega}{(1-\Omega)}\left(f_X+L^{*\gamma}\right)-\frac{\Omega^*}{(1-\Omega^*)}\left(f_X+L^\gamma\right)\right)>0.$$

Suppose that $L = L^*$. Then we know that the equation above is equal to zero. However, suppose that $L^* > L$. We know that

$$\beta\left(L^{*\gamma}-L^\gamma\right)>0.$$

But we do not know the sign of the rest of the equation. Is it positive?

$$\frac{\Omega}{(1-\Omega)}\left(f_X+L^{*\gamma}\right)>\frac{\Omega^*}{(1-\Omega^*)}\left(f_X+L^\gamma\right)\ ?$$

$$\begin{aligned}
\frac{\frac{(f_X+L^\gamma)^{-\beta}}{(f_D+L^\gamma)^{1-\beta}}}{(1-\Omega)} &> \frac{\frac{(f_X+L^{*\gamma})^{-\beta}}{(f_D+L^{*\gamma})^{1-\beta}}}{(1-\Omega^*)}\\
\frac{(f_X+L^\gamma)^{-1}}{\left((f_X+L^\gamma)^{\beta-1}(f_D+L^\gamma)^{1-\beta}-1\right)} &> \frac{\frac{(f_X+L^{*\gamma})^{-\beta}}{(f_D+L^{*\gamma})^{1-\beta}}}{\left(1-\frac{(f_X+L^{*\gamma})^{1-\beta}}{(f_D+L^{*\gamma})^{1-\beta}}\right)}
\end{aligned}$$

$$\begin{aligned}
\beta\left(L^{*\gamma}-L^\gamma\right)+(\beta-1)\left(\frac{\Omega}{(1-\Omega)}\left(f_X+L^{*\gamma}\right)-\frac{\Omega^*}{(1-\Omega^*)}\left(f_X+L^\gamma\right)\right) &> 0\\
\beta L^{*\gamma}+(\beta-1)\frac{\Omega}{(1-\Omega)}\left(f_X+L^{*\gamma}\right) &> \beta L^\gamma+(\beta-1)\frac{\Omega^*}{(1-\Omega^*)}\left(f_X+L^\gamma\right)
\end{aligned}$$

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