Protection for Sale to Oligopolists*

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Abstract

This paper modifies Grossman and Helpman's canonical "Protection for Sale" model by allowing demand linkages and oligopolistic competition. It shows that increased substitutability between products weakens interest groups' incentives to lobby. For the case of one organized and one unorganized industry, we obtain a particularly simple result: as product substitutability increases, the protection of the organized industry's product falls, whereas the protection of the unorganized sector's product increases.

Empirical studies of the "Protection for Sale" model have suggested that the U.S. government's trade policy decisions are overwhelmingly determined by a concern for welfare maximization; the alternative interpretation of the paper is that the original model overstates the lobby groups' desire for protection.

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1 Introduction

Recent theories of endogenous trade policy suggest that trade policy is determined through interaction between organized lobby groups and privately interested policy-makers, rather than by a benevolent social-welfare maximizer. While there are many variations on the theme (see Rodrik (1995) for a review), Grossman and Helpman's (1994) "Protection for Sale" model has been the most influential by far. Grossman and Helpman (henceforth GH) explicitly describe a mechanism through which interest groups' contributions influence the policy-maker's decision for trade protection, providing micro-foundations for the previous approaches. The model's predictions for the equilibrium protection pattern relate the industry's trade tariff to import demand elasticity, state of organization, import penetration and other variables, thereby providing a coherent framework for empirical testing.

As is well known, GH neglect some important issues. In particular, they abstract from production linkages and strategic market interactions. Indeed, factor-specific production in GH implies that different lobbies do not compete in the factor market. Hence, their interests are only opposed to the extent that each industry wants to increase its profits by raising the price of its own good, while all other organized industries aim at reducing the same price in order to increase their members' consumer surplus. That is, political competition among the lobbies arises purely from the desire of the members of different industry groups to protect their interests as ordinary consumers. This feature of GH has caused concerns about the interpretation of the model.[1] To add a more realistic justification for the political rivalry among the organized groups, Gawande and Bandyopadhyay (2000) introduce supply-side interactions through a single importable intermediate input. Cadot, de Melo and Ollarreaga (2001) strengthen the degree of inter-industry interactions even further, assuming that the industries compete for a common scare resource and each good is produced using other goods as intermediate inputs. These extensions help obtain predictions consistent with the observed stylized facts, e.g., escalation of protection rates with the degree of processing, or higher average protection for poor countries.

This paper instead focuses on demand-side interactions, addressing the rivalry between the organized groups that arises due to the substitutability between goods. It studies the impact of demand linkages on the determination of trade policy, and the intensity of inter-industry lobbying competition.

The consumers' utility function in the original GH model is assumed to be quasi-linear, implying that the demand for each product is independent of the prices of the other products. Besides, the GH model considers a small and open economy, so that the producers face perfectly elastic demand. Thus, the prices in other industries do not affect the producers' incentives to lobby. To address the inter-industry competition in lobbying and its impact on the trade policy, resulting from the demand-side interactions, I relax these assumptions and allow for both demand linkages and imperfectly competitive industries. More precisely, I consider a utility function that admits cross-price effects on demand and assume that each good is produced by an international oligopoly and sold in internationally segmented markets.

The first part of the paper shows that the presence of substitutes may reduce interest

[1] See e.g. Baldwin and Robert-Nicoud (2006).

group incentives to lobby due to competition in the goods market. If demands are interdependent, an increase in the price of a good causes demand to shift towards its substitutes. A decrease in the price of the substitute has a similar effect. The interest group takes this shift into account when lobbying the government to increase the price of its own good (as a producer receiving profit from selling this good) and reduce the price of all other goods (as a consumer who wants to maximize her utility). Therefore, the lobbying strategy of an organized industry becomes less aggressive. To put it differently, introducing substitutability produces more aligned interests between the interest groups. As a result, other things equal, economies producing closer substitutes (or having a more competitive industry structure) should experience more moderate protection rates. That is, with an increase in the degree of substitutability, the protection of the organized industries falls, and the protection of the non-organized industries increases, relative to the first-best benchmark.

The result suggests a new explanation for an observed empirical puzzle. Studies devoted to empirical testing of the GH theory find that the government puts very low weight on the campaign contributions relative to the welfare loss. That is, the government behaves almost as a social welfare maximizer and trade policy deviates surprisingly little from the first-best,[2] thus causing a concern about the empirical significance of the GH model. E.g., Gawande and Krishna (2002) write that "..it is enough to cast doubt on the value of viewing trade policy determination through this political economy lens". There have been several attempts at modifying the model to explain low protection rates. For example, Gawande and Bandyopadhyay (2000) introduce political competition between the upstream and downstream producers, and Gawande, Krishna and Robbins (2004) introduce counter-lobbying by the foreign organized groups. This paper provides an alternative explanation, suggesting that smaller deviations from the first-best protection rates may result from the weaker incentives to lobby caused by the substitutability effects.

The second part of the paper addresses the impact of product substitutability on lobby formation. The original GH model assumes an exogenous lobby group structure. A natural question to ask is thus why are some industries organized and others not. In the GH model, the interests of different industry groups are opposed to each other, so if an additional lobby formation stage is introduced in their model, all industries would get organized (in the absence of the lobby formation costs).[3]

However, in the presence of demand linkages, at a sufficiently high degree of substitution a non-organized industry becomes protected without paying for it. The reason is that the organized industry cannot lobby to increase its own price without losing a substantial part of its consumers switching to the cheaper (but similar) good. For the same reason, it cannot lobby for dropping the price of the substitute. Thus, the non-lobbying industry gets a free ride on the lobbying industry efforts. If instead both these substitute-producing industries are organized, they both contribute to the government for (potentially higher) protection. Comparing these two outcomes, it turns out that the industry may better off not being organized. As a result, with endogenous lobby formation, fewer industries get organized and lobbying becomes less intense. We find this free-riding effect to be present for industries with relatively dispersed ownership, which is in line with Olson's (1965) interest group size

[2] E.g. Goldberg and Maggi (1999); Gawande and Bandyopadhyay (2000).

[3] Like GH, we neglect any intra-industry organizational conflict.

hypothesis.

This paper is not the first to endogenize lobby formation in the GH model or discuss the possibility of free-riding. Mitra (1999) adds an initial stage to the GH model, letting the owners of each specific factor decide in Nash equilibrium whether it is profitable to incur a fixed cost of forming a lobby. Magee (2002) employs a two-stage game, where in the first stage, industry representatives and the policy maker determine the tariff schedule and, in the second stage, every firm in the industry decides whether to contribute to the lobbying effort (detecting is infinitely punished). Both these papers address the collective action problem at the intra-industry level. We, instead, relate lobby formation to the inter-industry demand links, thereby avoiding the issue of exogenous fixed costs.

The remainder of the paper is organized as follows: Section 2 describes the model setup, Section 3 discusses the equilibrium structure of protection, Section 4 analyzes the effect of the degree of substitution on the extent of protection, Section 5 studies the question of the lobby formation in the presence of substitute goods and Section 6 concludes.

2 The model

There are $m+1$ goods being produced in an open economy. The domestic price of good i is denoted p_i. Good 0 is taken to be a numeraire, so $p_0 = 1$. The individuals populating the economy have identical preferences represented by the quasi-linear utility function

$$U(x_0, x_1, ..., x_m) = x_0 + \hat{U}(x_1, ..., x_m), \tag{1}$$

where x_i denotes consumption of good i. The original GH model uses the separable utility function

$$U(x_0, x_1, ..., x_n) = x_0 + \sum_{i=1}^{n} \hat{u}_i(x_1, ..., x_n),$$

which implies that demand functions are independent of the prices of the other goods. I relax this assumption allowing for the cross-price effects. More precisely, I adopt the quadratic sub-utility function:

$$\hat{U}(x_1, ..., x_m) = \sum_{k=1}^{m} x_k - 1/2 \sum_{k=1}^{m} x_k^2 - \sigma \sum_{k=1}^{m} \sum_{j=k+1}^{m} x_k x_j,$$

where $\sigma \in [0, 1]$ reflects the substitutability between goods $x_1, ...x_m$. As is well known, the associated inverse demand function for each non-numeraire good i is

$$p_i = 1 - x_i - \sigma \sum_{j \neq i}^{m} x_j.$$

For $\sigma \in (0, 1)$, considering only interior solutions, the resulting domestic demand function for good i is

$$d_i(p_1, ..., p_m) = \frac{(1-\sigma) - ((m-2)\sigma + 1)\, p_k + \sigma \sum_{j \neq k}^{m} p_j}{((m-1)\sigma + 1)\,(1-\sigma)}.$$

In words, domestic demand is linear in prices of non-numeraire goods, decreasing in the price of the own good and increasing in the prices of the other goods. That is, good i and goods $1, 2..i-1, i+1, ..m$ are imperfect substitutes.

Due to the quasi-linearity of the utility function, any income effect is totally captured by the consumption of good 0. That is, when optimally spending an amount E, each individual obtains demand functions

$$x_i = d_i(\mathbf{p}),\ i \in \{1, ..., m\}$$
$$x_0 = E - \sum_{k=1}^{m} p_k d_k(\mathbf{p}),$$

where $\mathbf{p}$ is the vector of domestic prices $(p_1, ..., p_m)$. The associated indirect utility function is

$$V(\mathbf{p}) = E - \sum_{k=1}^{m} p_k d_k(\mathbf{p}) + \hat{U}(d_1(\mathbf{p}), ..., d_m(\mathbf{p})).$$

The numeraire good is produced using labor only, with an input-output coefficient of 1. As the non-numeraire good is freely traded in a perfectly competitive international market, the wage rate in this economy is equal to 1. The other m goods are produced by a CRS technology using labor alone, but are sold at internationally segmented oligopolistic markets. More precisely, good k is supplied by $n_k > 0$ identical domestic firms and $n_k^* > 0$ identical international firms, competing in quantities. Thus, the total number of firms operating in sector k is given by

$$N_k = n_k + n_k^*.$$

It takes c_k units of labor to produce one unit of good k, both at home and abroad.[4] We assume that each firm is a pure profit maximizer and that the number of firms is fixed, so that no entry or exit decisions are taken.

Each individual is endowed with some labor and may also own some claims to the profit of a firm in at most one industry. These claims are indivisible and non-tradable.

The government introduces trade taxes and/or subsidies and redistributes the resulting net revenue from all these taxes equally among the voting population. For each non-numeraire good, the domestic and the international market are segmented and production is characterized by the constant marginal costs, c_k. As a result, firms' production decisions are made separately for the domestic and the international market. The trade tax revenue is separable in the import and export tax and, as a result, so is consumer welfare. Thus, the government can set import and export trade policies separately. In this paper, we concentrate on the government decision about import tariffs, which determines the production and consumption in the domestic market. We also assume that the foreign government does not impose any export tariffs on its firm, so that in the absence of trade intervention of the domestic government, firms at home and abroad are equally efficient. As we do not study the strategic interaction between the home government and foreign governments in setting trade policy, this assumption does not affect the model's qualitative results.

[4] The assumption that the domestic and the foreign firm have the same unit cost c_k is made for computational convenience. It does not change the predictions of the model.

We denote the trade tariff in sector k by τ_k. Then, in Cournot-Nash equilibrium, each of the n_k domestic firms in sector k solves the profit maximization problem

$$\max_{q_k^i} \pi_k^i = q_k^i \left[1 - \sum_{j=1}^{n_k} q_k^j - \sum_{j=1}^{n_k^*} q_k^{j*} - \sigma \sum_{s \neq k} \left(\sum_{j=1}^{n_s} q_s^j + \sum_{j=1}^{n_s^*} q_s^{j*} \right) \right] - c_k q_k^i, \tag{2}$$

where q_k^i denotes the production of domestic firm i in sector k, and q_k^{j*} denotes the production of foreign firm i in sector k. Similarly, each of the n_k^* foreign firms in sector k solves the problem

$$\max_{q_k^{i*}} \pi_k^{i*} = q_k^{i*} \left[1 - \sum_{j=1}^{n_k} q_k^j - \sum_{j=1}^{n_k^*} q_k^{j*} - \sigma \sum_{s \neq k} \left(\sum_{j=1}^{n_s} q_s^j + \sum_{j=1}^{n_s^*} q_s^{j*} \right) \right] - (c_k + \tau_k)\, q_k^{j^*}. \tag{3}$$

Solving the system of these equations for all sectors yields the equilibrium quantities produced by each firm. The symmetry assumption implies that firms within each sector choose the same equilibrium output at home,

$$q_k^i(\boldsymbol{\tau}) = q_k^j(\boldsymbol{\tau}) \equiv q_k(\boldsymbol{\tau}),$$

and abroad

$$q_k^{i*}(\boldsymbol{\tau}) = q_k^{j*}(\boldsymbol{\tau}) \equiv q_k^*(\boldsymbol{\tau}),$$

where $\boldsymbol{\tau}$ denotes the vector of the trade tariffs $(\tau_1, ...\tau_m)$. Similarly, in industry k**,** each firm in the same country earns the same profits, $\pi_k(\boldsymbol{\tau})$ for a domestic firm and $\pi_k^*(\boldsymbol{\tau})$ for a foreign firm, respectively. Moreover, from the first-order conditions of the profit maximization problems for the domestic firm (2) and the foreign firm (3) in sector k, it follows that the profit is given by

$$\begin{aligned} \pi_k(\boldsymbol{\tau}) &= q_k^2, \\ \pi_k^*(\boldsymbol{\tau}) &= (q_k^*)^2 . \end{aligned} \tag{4}$$

Note also that the market clearing condition implies that

$$d_k(\mathbf{p}(\boldsymbol{\tau})) = n_k q_k(\boldsymbol{\tau}) + n_k^* q_k^*(\boldsymbol{\tau}).$$

The amount of the tax revenue collected by the government (and redistributed to the citizens) is equal to

$$r(\boldsymbol{\tau}) = \sum_{k=1}^{m} \tau_k n_k^* q_k^*(\boldsymbol{\tau}). \tag{5}$$

Thus, individual income is the sum of wages, government transfers and possibly claims to a domestic firm's profit.

The owners of firms in the same industry may choose to organize and form a lobby group trying to influence the government in its decision about trade policies. The joint welfare of the members of such a group i comprising share α_i of the total population is

$$W_i(\boldsymbol{\tau}) = l_i + n_i \pi_i(\boldsymbol{\tau}) + \alpha_i \left[r(\boldsymbol{\tau}) - \sum_{k=1}^{m} p_k d_k(\mathbf{p}) + \hat{U}(d_1(\mathbf{p}(\boldsymbol{\tau})), ..., d_m(\mathbf{p}(\boldsymbol{\tau}))) \right], \tag{6}$$

where l_i denotes the total labor endowment of group i members. Note that the vector of equilibrium prices is, in turn, a function of trade tariffs.

Each lobby i may contribute to the government an amount $C_i(\boldsymbol{\tau})$ conditional on the trade policy vector, or, equivalently, the domestic price vector.

The objective function of the government is

$$G(\boldsymbol{\tau}) = \sum_{i \in L} C_i(\boldsymbol{\tau}) + aW(\boldsymbol{\tau}),$$

where L is the exogenously given set of organized sectors, $a > 0$ is the weight the government attaches to aggregate welfare in the economy, and

$$W(\boldsymbol{\tau}) = l + \sum_{k=1}^{m} n_k \pi_k(\boldsymbol{\tau}) + \left[r(\boldsymbol{\tau}) - \sum_{k=1}^{m} p_k d_k(\mathbf{p}(\boldsymbol{\tau})) + \hat{U}(d_1(\mathbf{p}(\boldsymbol{\tau})), ..., d_m(\mathbf{p}(\boldsymbol{\tau}))) \right] \tag{7}$$

is aggregate welfare in the economy.

Trade policy is determined in a two-stage game. In the first stage, lobbies simultaneously announce their contribution schedules, i.e., the amount contributed as a function of the import tariffs vector (note that *each* strategy of a lobby is a *function* on the price simplex). In the second stage, the government chooses policy by maximizing its objective function over the suggested contribution schedules.

The resulting equilibrium is a set of contribution functions, each maximizing the welfare of the respective lobby members given the schedules of all other lobbies and the anticipated tariff decision of the government and the price vector maximizing the government's objective function under these contribution schedules.

3 The structure of protection

The problem has the structure of a menu-auction, as characterized by Bernheim and Whinston (1986). Hence, if contribution schedules are differentiable, they are *locally truthful* around equilibrium, i.e., a marginal change in each lobby's contribution to the government resulting from a small policy change is exactly equal to the respective marginal change of this lobby's welfare.

Local truthfulness combined with the optimality of the equilibrium price vector for the government results in the condition (equation (12) in GH)

$$\sum_{i \in L} \nabla W_i(\boldsymbol{\tau}) + a \nabla W(\boldsymbol{\tau}) = \mathbf{0}. \tag{8}$$

Substituting the tariff revenue (5) and the expression for lobby i's welfare (6) into (8) yields

$$(a + \alpha_L) \nabla \left[\sum_{k=1}^{m} \tau_k n_k^* q_k^*(\boldsymbol{\tau}) - \sum_{k=1}^{m} p_k d_k(\mathbf{p}(\boldsymbol{\tau})) + \hat{U}(d_1(\mathbf{p}(\boldsymbol{\tau})), .., d_m(\mathbf{p}(\boldsymbol{\tau}))) \right] \tag{9}$$

$$+ \sum_{k=1}^{m} (I_k + a) n_k \nabla \pi_k(\boldsymbol{\tau}) = \mathbf{0},$$

where I_k is an indicator function taking the value of 1 if industry k is organized, and 0 otherwise, and $\alpha_L = \sum_{i \in L} \alpha_i$ is the total share of population in the organized industries. Simplifying the system (9) (the detailed derivation can be found in the Appendix) entails the following form for each equation j in (9):

$$-\sum_{k=1}^{m} \tau_k \frac{\partial m_k(\boldsymbol{\tau})}{\partial \tau_j} = \sum_{k=1}^{m} \frac{(I_k + a)}{(a + \alpha_L)} n_k \frac{\partial \pi_k(\boldsymbol{\tau})}{\partial \tau_j} - \sum_{k=1}^{m} \frac{\partial p_k}{\partial \tau_j} d_k(\mathbf{p}(\boldsymbol{\tau})) + n_j^* q_j^*(\boldsymbol{\tau}), \tag{10}$$

where

$$m_k(\boldsymbol{\tau}) = n_k^* q_k^*(\boldsymbol{\tau})$$

denotes total imports in sector k. In what follows, we shall study the properties of the system of equations (10).

Before proceeding, we need to establish several intermediate results concerning the responsiveness of domestic and foreign supply to the trade tariffs.

Lemma 1 *The equilibrium quantity produced by the foreign firm in sector k decreases in the trade tariff on its own good, τ_k, and weakly increases in the trade tariff on the substitute goods, τ_{-k}. That is, $\partial q_k^*(\boldsymbol{\tau})/\partial \tau_j < 0$ if $i = k$ and $\partial q_k^*(\boldsymbol{\tau})/\partial \tau_j \geq 0$ if $i \neq k$.*
The equilibrium quantity produced by the domestic firm in sector k increases in the trade tariff on each good $i = 1, ..., m$. That is, $\partial q_k(\boldsymbol{\tau})/\partial \tau_j > 0$ if $i = k$ and $\partial q_k(\boldsymbol{\tau})/\partial \tau_j \geq 0$ if $i \neq k$.

Proof. See the Appendix. ■

Lemma 1 follows from two observations. First, consider the own-tariff effect. Due to Cournot competition and linear demand functions, the home and foreign outputs in sector k are strategic substitutes. A domestic import tariff in sector k shifts the foreign firm's response curve in the south-west direction. Therefore, an increase in τ_k reduces the output of the foreign firm and increases the output of the domestic firm in sector k. Now, turn to the cross-tariff effect. The output produced by either a domestic or a foreign firm in sector k and the output produced by a foreign firm in sector j are strategic substitutes. As a result, an increase in τ_j induces higher output of both domestic and foreign firms in sector k.

As the domestic profits are given by relation (4), we immediately obtain the following corollary.

Corollary 2 *The equilibrium profit of the domestic firm in sector k increases in the trade tariff on each good $i = 1, ..., m$. That is, $\partial \pi_k(\boldsymbol{\tau})/\partial \tau_j > 0$ for all k, j.*

Let us now turn to the analysis of equation (10). First, consider the case of a separable utility function, so that $\sigma = 0$.[5] This immediately implies that $\partial m_k(\boldsymbol{\tau})/\partial \tau_j = \partial \pi_k(\boldsymbol{\tau})/\partial \tau_j = \partial p_k/\partial \tau_j = 0$ for $k \neq j$. In this case, equation (10) becomes

$$\tau_j = \frac{1}{(-\partial m_j(\boldsymbol{\tau})/\partial \tau_j)} \left[\frac{(I_j + a)}{(a + \alpha_L)} n_j \frac{\partial \pi_j(\boldsymbol{\tau})}{\partial \tau_j} - \left(\frac{\partial p_j}{\partial \tau_j} d_j(\mathbf{p}(\boldsymbol{\tau})) - n_j^* q_j^*(\boldsymbol{\tau}) \right) \right], \tag{11}$$[6]

[5] It parallels the original GH setting in **an** imperfectly competitive environment.
[6] It is equivalent to

$$\frac{\tau_k}{p_k} = \frac{(2I_k + 2a)}{(a + \alpha_L)} \frac{X_k(\boldsymbol{\tau})}{m_k(\boldsymbol{\tau})} \frac{1}{\left| \frac{\partial m_k(\boldsymbol{\tau})}{\partial \tau_k} / \frac{m_k(\boldsymbol{\tau})}{p_k} \right|} + \frac{1}{p_k \frac{\partial m_k(\boldsymbol{\tau})}{\partial \tau_k}} \left(\frac{\partial p_k}{\partial \tau_k} d_k(\mathbf{p}(\boldsymbol{\tau})) - n_k^* q_k^*(\boldsymbol{\tau}) \right),$$

and in the absence of lobbying, the first-best trade tariffs are determined by

$$\tau_j^0 = \frac{1}{(-\partial m_j(\boldsymbol{\tau})/\partial \tau_j)} \left[n_j \frac{\partial \pi_j(\boldsymbol{\tau})}{\partial \tau_j} - \left(\frac{\partial p_j}{\partial \tau_j} d_j(\mathbf{p}(\boldsymbol{\tau})) - n_j^* q_j^*(\boldsymbol{\tau}) \right) \right]. \tag{12}$$

Due to imperfect competition in the goods markets, free trade is no longer socially optimal. But, as in the original GH model, the organized industries experience higher protection than in the first-best equilibrium. In turn, the non-organized industries are underprotected. Indeed, other things equal, equations (11) and (12) differ by the term

$$\frac{1}{(-\partial m_j(\boldsymbol{\tau})/\partial \tau_j)} n_j \frac{\partial \pi_j(\boldsymbol{\tau})}{\partial \tau_j}, \tag{13}$$

having the factor $(1+a)/(a+\alpha_L)$ in the latter equation. From Lemma 1, it follows that $-\partial m_j(\tau)/\partial \tau_j > 0$, $\partial \pi_j(\tau)/\partial \tau_j > 0$ and thus, the entire term (13) is positive. As long as the organized industries do not comprise the entire population ($\alpha_L < 1$), the factor $(I_j + a)\,/\,(a + \alpha_L)$ exceeds 1 for an organized industry ($I_j = 1$) but falls below 1 for an unorganized industry ($I_j = 0$). Therefore, other things equal, for an organized industry the tariff is higher than the first-best tariff. Similarly, for a non-organized industry the tariff set in the presence of lobby groups is lower than the first-best tariff. Note that if all industries are organized and every voter belongs to some lobby ($\alpha_L = I_j = 1\ \forall j$), the protection rates are equal to the first-best ones. That is, like in GH, the lobbying efforts of different industries exactly offset each other.

Now, let us see how the relaxation of utility separability influences the equilibrium tariffs. Once more, we start by establishing an auxiliary result – we want to characterize the matrix of the derivatives of the import with respect to the trade tariffs. Denote it by M

$$M = (M_{kj}) = \left(\frac{\partial m_k}{\partial \tau_j} \right), \quad 1 \le k, j \le N.$$

Lemma 3 *The matrix M is invertible and its inverse M^{-1} is non-positive, i.e., all entries of M^{-1} are non-positive.*

Proof. See the Appendix. ■

We use Lemma 3 to obtain the expression for the trade tariffs from the system (10). System (10) in matrix form becomes

$$\begin{aligned} -M\boldsymbol{\tau} &= -\left(\frac{\partial m_k}{\partial \tau_j} \right) (\tau_j) \\ &= \left(\sum_{k=1}^{m} \frac{(I_k + a)}{(a + \alpha_L)} n_k \frac{\partial \pi_k(\boldsymbol{\tau})}{\partial \tau_j} - \sum_{k=1}^{m} \frac{\partial p_k}{\partial \tau_j} d_k(\mathbf{p}(\boldsymbol{\tau})) + n_j^* q_j^*(\boldsymbol{\tau}) \right). \end{aligned} \tag{14}$$

Denote the negative of the matrix M^{-1} by

$$B = -(M)^{-1} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mm} \end{pmatrix}.$$

where $X_k(\boldsymbol{\tau})$ denotes total home production of good k, $X_k(\tau) = n_k q_k(\tau)$. That is, we replicate the result (12) in Gawande, Krishna and Robbins (2004), up to the foreign lobbies component and the approximation they use ($\frac{\partial p_k}{\partial \tau_k} d_k(\mathbf{p}(\boldsymbol{\tau})) \approx n_k^* q_k^*(\boldsymbol{\tau})$).

Lemma 3 implies that each element of B is nonnegative, $b_{ij} \geq 0$. Multiplying equation (14) by B yields

$$\tau_i = \sum_{j=1}^{m} b_{ij} \left(\sum_{k=1}^{m} \frac{(I_k + a)}{(a + \alpha_L)} n_k \frac{\partial \pi_k(\boldsymbol{\tau})}{\partial \tau_j} - \sum_{k=1}^{m} \frac{\partial p_k}{\partial \tau_j} d_k(\mathbf{p}(\boldsymbol{\tau})) + n_j^* q_j^*(\boldsymbol{\tau}) \right), \tag{15}$$

which is equivalent to the system (10).

Let us analyze the system (15). First, we note that the first-best trade tariffs are given by equation

$$\tau_i^0 = \sum_{j=1}^{m} b_{ij} \left(\sum_{k=1}^{m} n_k \frac{\partial \pi_k(\boldsymbol{\tau})}{\partial \tau_j} - \sum_{k=1}^{m} \frac{\partial p_k}{\partial \tau_j} d_k(\mathbf{p}(\boldsymbol{\tau})) + n_j^* q_j^*(\boldsymbol{\tau}) \right).$$

As above, we find that if all industries are organized and every voter belongs to some lobby ($\alpha_L = I_j = 1\ \forall j$), the equilibrium protection rates are first-best.

However, unlike the case with a separable utility (system (11)), it does not imply that the non-organized industries are always underprotected and the organized industries are overprotected. Indeed, we see that in the presence of substitutability between products, the trade tariffs are directly affected by the sensitivity of supply and demand in the other industries and the organizational status of these industries. That is, the terms corresponding to the reaction of the other industries to the increase in τ_i, directly appear in the tariff equation for industry i. In particular, the negative effect of industry k being organized on industry i's protection is weaker if industries i and k produce substitutes. To see this, compare equations (11) and (15) for industry i. First, assume that the goods are independent ($\sigma = 0$), so that the tariff for industry i is given by equation (11) for $j = i$. Other things equal, industry k getting organized reduces the protection for industry i only through an increase in α_L (as the corresponding factor $(I_i + a)/(a + \alpha_L)$ decreases and $n_i\,(\partial \pi_i(\boldsymbol{\tau})/\partial \tau_i) > 0$). Now, instead, consider the case when the goods are substitutes and the tariff for industry i is given by equation (15)). As above, industry k getting organized reduces the industry i tariff through higher α_L in factors $(I_s + a)/(a + \alpha_L)$ for all $s \in \{1, ..., m\}$, as $\partial \pi_s(\tau)/\partial \tau_j \geq 0$ and $b_{ij} \geq 0$ for all i, j. However, it also has a positive effect on industry i's protection through an increase in I_k from 0 to 1 in factor $(I_k + a)/(a + \alpha_L)$. It is unclear which effect dominates, but this argument suggests that the overall negative impact on protection of good i resulting from industry k being organized is weaker in case of substitutability between the goods.

The intuition behind this effect is straightforward: in the absence of substitution, that is, if the utility function is quasi-linear and separable in goods $i = 1, ..., m$, the consumption of each non-numeraire good is fully determined by the price of this good only. As a result, each organized group has two goals. First, it tries to raise the trade tariff in its own sector, which increases its market share, the own good price and, thus, the profit of the lobby. At the same time, it attempts to reduce the tariffs on all other goods it consumes, as lower tariffs entail lower consumption prices. However, in the presence of substitution between the goods, such a lobbying strategy may cause consumers to switch consumption from the most highly protected goods to less protected ones. To limit substitution, organized industries tend to apply more "moderate" lobbying strategies. That is, they try to maintain a balance between decreasing the price of the other goods and increasing its own price.

This effect is best understood in case the ownership of industry k is very highly concentrated, that is, the share α_k of the population entitled to its profit is zero. Then, the total share of the population in the organized industries α_L does not change when industry k becomes organized. In this case, the effect of other industries' price change on lobby k's welfare is negligible, so that they have no consumer welfare gain from the trade interventions in other sectors. Therefore, in the absence of substitutability, if they get organized, it does not influence the protection of any other industry $j \neq k$ (indeed, as α_L does not change, equations (11) for tariffs in industries $j \neq k$ are unaffected). However, with substitutability between the goods, industry k still lobbies for additional protection of all substitute-producing industries because it is concerned with maintaining demand for its own good. That is, it increases industry i protection, as the term

$$\sum_{j=1}^{m} b_{ij} \frac{I_k}{(a+\alpha_L)} n_k \frac{\partial \pi_k(\boldsymbol{\tau})}{\partial \tau_j} = \sum_{j=1}^{m} b_{ij} \frac{1}{(a+\alpha_L)} n_k \frac{\partial \pi_k(\boldsymbol{\tau})}{\partial \tau_j}$$

entering the equation for the equilibrium tariff τ_i is positive.

In GH, the interests of different industry groups are opposed to each other. Here, we see that non-organized industries may benefit from the contributions of organized ones by "exploiting" the demand properties. We return to this discussion in the subsequent sections and show that this effect caused by substitutability of products can lead to free-riding in the lobbying behavior.

The above results are obtained for Cournot competition. However, it is easily seen that they also hold if we keep the assumption of linear demand and constant marginal costs, but allow firms to compete in prices in a Bertrand-fashion. Also in this case does a higher tariff in industry i convert into higher prices and demand shifting away from sector i, while higher tariffs in other sectors attract more demand into sector i. Indeed, prices are strategic compliments and therefore, they increase in each sector's tariffs. From the profit maximization problem it follows that, in equilibrium, higher domestic prices entail higher domestic outputs

$$q_i = (p_i - c_i)\left(-\frac{\partial q_i}{\partial p_i}\right),$$

and the same is true for foreign output in sector i for a given tariff τ_i

$$q_i^* = (p_i - c_i - \tau_i)\left(-\frac{\partial q_i}{\partial p_i}\right). \tag{16}$$

Therefore, domestic output increases in each sector's tariff and foreign output increases in all other sectors' tariffs. As the sensitivity of the foreign price with respect to the own tariff is never above one (an increase in industry i's tariff is fully captured by an increase in price only in case domestic and foreign goods in industry i are perfect substitutes), formula (16) implies that foreign output is decreasing in its own tariff. It can be shown that under reasonable assumptions, the corresponding matrix M is invertible and thus the qualitative results do not change.

4 The level of protection

We are not ready to investigate whether, all else equal, more substitutability in consumer preferences causes less protection. To make the analysis tractable, we assume that there is only one domestic and one foreign firm operating in each imperfectly competitive sector, so that the respective non-numeraire good k is supplied by a duopoly, i.e.,

$$\begin{aligned} n_k &= n_k^* = 1, \\ N_k &= 2. \end{aligned}$$

Furthermore, we limit ourselves to the case of two non-numeraire goods, $m = 2$, and assume the marginal costs of production to be equal across sectors, $c_1 = c_2$. We concentrate on the interior solutions.[7]

In this case, the profit maximization problem of each domestic firm takes the form

$$\max_{q_k} \pi_k = q_k \left[1 - (q_k + q_k^*) - \sigma \left(q_{-k} + q_{-k}^*\right)\right] - c q_k, \quad k = 1, 2.$$

Similarly, each foreign firm solves

$$\max_{q_k} \pi_k^* = q_k^* \left[1 - (q_k + q_k^*) - \sigma \left(q_{-k} + q_{-k}^*\right)\right] - (c + \tau_k) q_k, \quad k = 1, 2.$$

>From the first-order conditions, it follows that for a given vector of trade tariffs $\boldsymbol{\tau} = (\tau_1, \tau_2)$, in an interior Nash equilibrium each domestic firm produces

$$q_k = \frac{(1-c)(3-2\sigma) + \left(3 - 2\sigma^2\right)\tau_k + \sigma\tau_{-k}}{9 - 4\sigma^2}, \tag{17}$$

while each foreign firm produces

$$q_k^* = \frac{(1-c)(3-2\sigma) - 2\left(3 - \sigma^2\right)\tau_k + \sigma\tau_{-k}}{9 - 4\sigma^2}. \tag{18}$$

The profit in each case is determined by equality (4).

Substituting expressions (17), (18) and (4) into the system (10), we can solve for the equilibrium trade tariffs. If none of the industries is organized, so that $\alpha_L = I_1 = I_2 = 0$, the government implements the first-best policy and imposes import taxes

$$\tau_1^0(\sigma) = \tau_2^0(\sigma) = \frac{(1-c)}{\sigma + 3}. \tag{19}$$

Now, consider the case when only industry 1 is organized and it represents a proportion α of the total population. The resulting trade tariff for industry 1 is given by

$$\begin{aligned} \tau_1(\sigma, \alpha, a) = {} & (1-c)\left[4(a+1)(a+2\alpha)\sigma^3 + 4\left(\alpha^2 - 3a\alpha - 2a - 3a^2 - 3\alpha\right)\sigma^2\right. \\ & - \left(9a^2 + 26a\alpha + 10a + 13\alpha^2 + 14\alpha\right)\sigma \\ & \left. + (3a + \alpha + 2)(9a + 11\alpha)\right] * \left[4(a + 2\alpha - 1)(a + 2\alpha)\sigma^4\right. \\ & + \left(14a - 118a\alpha - 45a^2 - 81\alpha^2 + 22\alpha\right)\sigma^2 \\ & \left. + (9a + 11\alpha - 2)(9a + 11\alpha)\right]^{-1}, \end{aligned} \tag{20}$$

[7] It will be shown in the proof of Proposition 4 that a necessary condition for an interior solution is $a + 3\alpha \geq 2$.

and the tariff for industry 2 is determined by

$$\begin{aligned}\tau_2(\sigma,\alpha,a) = {} & (1-c)\left[4a(a+2\alpha-1)\sigma^3 - 4\left(a+2\alpha+3a\alpha-\alpha^2+3a^2\right)\sigma^2\right.\\ & +(18a-26a\alpha-9a^2-13\alpha^2+14\alpha)\sigma\\ & +(3a+\alpha)(9a+11\alpha-2)]*\left[4(a+2\alpha-1)(a+2\alpha)\sigma^4\right.\\ & +\left(14a-118a\alpha-45a^2-81\alpha^2+22\alpha\right)\sigma^2\\ & +(9a+11\alpha-2)(9a+11\alpha)]^{-1},\end{aligned} \tag{21}$$

(see the Appendix for the derivation of equations (19), (20) and (21)).

We would like to show that the increase in the degree of competition in the product market (represented by the increase in the degree of substitution between the products) reduces the incentives for the lobby to raise its own price and/or lower the prices of the other goods. Hence, it results in a more moderate equilibrium trade protection for the organized interest group.

As shown by equation (19), an increase in the degree of substitution decreases the protection level even in the absence of lobbying – the first-best trade tariffs decline in σ. The intuition is as follows: A domestic import tax in sector i is aimed at increasing the market share of the domestic firm.[8] As the degree of substitutability increases, the effect of the trade tax in sector i also extends to the firms in sector $-i$, thereby increasing their output and improving their market position. Hence, due to the competition arising from substitutability, the overall effect of a tax in sector i on the firm in sector i becomes weaker, leaving less room for strategic trade policy. When examining the influence of the degree of substitutability on lobbying, we want to abstract from this effect. We do it by analyzing how the trade tariffs in a lobbying equilibrium differ from the first-best trade policy. More precisely, we study the ratios

$$T_i(\sigma,\alpha,a) = \tau_i(\sigma,\alpha,a)/\tau_i^0(\sigma), \quad i=1,2,$$

which we henceforth refer to as relative protection, and their response to the change in the degree of substitution σ.

Proposition 4 *If industry* 1 *is organized, while industry* 2 *is not, the equilibrium relative protection of the organized industry decreases with the degree of substitution:*

$$\frac{dT_1(\sigma,\alpha,a)}{d\sigma} < 0.$$

For the non-organized industry, the effect is the opposite – the equilibrium relative protection increases with the degree of substitution:

$$\frac{dT_2(\sigma,\alpha,a)}{d\sigma} > 0.$$

Proof. See the Appendix. ■

It is worth noting that the result of Proposition 4 does not necessarily imply that the influence-driven protection rates will converge to the socially-optimal level as the degree of substitutability increases. In fact, for a larger σ, the organized industry is interested

[8] Subject to the respective loss in consumer welfare and the change in the tax revenues.

in reducing the difference between the price of its own good and the substitute, not in achieving the first-best outcome. For $\sigma = 1$, the goods become indistinguishable from the consumer's point of view, i.e., industries 1 and 2 face a joint demand for these goods. Hence, the foreign firms and sector 1 and sector 2 become identical competitors from the point of view of the domestic organized industry 1, so it lobbies for the same trade tariff in both industries. However, depending on the concentration of industry 1 (that is, of the share of population $\alpha_1 = \alpha$ that has claims to its profit) this tariff can differ from the first-best level in either direction. If $\alpha = 1/2$, industry 1's members are exactly representative of the entire population – they own as much of the firm's profit claims [9] as does the average person. Therefore, the tariff for which the industry is lobbying perfectly matches the first-best tariff. If instead industry 1 is more concentrated, so that $\alpha < 1/2$, it cares about the domestic profits, as opposed to consumer welfare, more than does the average person. Therefore, in this case, the equilibrium protection rates exceed the socially optimal ones. Similarly, if $\alpha > 1/2$, the resulting equilibrium protection falls below the socially optimal level.

Still, if the degree of substitution is not too high, the non-organized industry is always underprotected as compared to the first-best trade tariff, because

$$T_2(0, \alpha, a) = 3\frac{(3a + \alpha)}{(9a + 11\alpha)} = 1 - 8\frac{\alpha}{9a + 11\alpha} < 1,$$

and the relative protection rate changes continuously. The organized industry achieves more than the first-best protection, as

$$T_1(0, \alpha, a) = 3\frac{(3a + \alpha + 2)}{(9a + 11\alpha - 2)} = 1 + \frac{8(1 - \alpha)}{9a + 11\alpha - 2} > 1.$$ [10]

Therefore, we have the following corollary:

Corollary 5 *If the degree of substitutability is not too high, an increase in σ shifts the influence-driven protection rates towards the socially optimal levels.*

We have shown above that competition in the lobbying market can reduce the deviation of the influence-based protection from the first-best level. For example, when every person owns shares of one or the other industry and both industries actively participate in lobbying, the economy ends up with the socially-optimal trade tariffs. Corollary 5 demonstrates that competitive pressure in the product market (resulting from the rise in the degree of substitution) can work in the same direction.

This result suggests a new explanation for a puzzle commonly observed in the empirical studies of GH theory. Most studies report extremely low estimates of the weight government puts on campaign contributions relative to social welfare.[11] That is, the government behaves almost as a social welfare maximizer and trade policy deviates surprisingly little from the first-best, which has caused a concern about the empirical significance of the GH model. For example, Gawande and Krishna (2001) write that "..it is enough to cast doubt on the value

[9] Note that in case of perfect substitutability, domestic firms in sectors 1 and 2 are exactly identical as are their profit functions.

[10] As shown in the proof of Proposition 4, the necessary condition for the interior solution is $a + 3\alpha > 2$, which implies that $9a + 11\alpha - 2 > 0$.

[11] See e.g. Goldberg and Maggi (1999) and Gawande and Bandyopadhyay (2000).

of viewing trade policy determination through this political economy lens". Recently, several papers have modified the GH model in order to resolve the puzzle. This research primarily emphasizes the idea of a lobbying competition. Gawande and Bandyopadhyay (2000) introduce political competition between the producers of intermediate and final goods, Gawande and Krishna (2005) go even further by bringing in the cross-sectoral connections through the input-output matrix, and Gawande, Krishna and Robbins (2004) introduce counter-lobbying by foreign organized groups. This paper provides an alternative explanation, suggesting that smaller deviations from the first-best trade policy may result from the weaker incentives to lobby, due to product substitutability. Indeed, all studies cited above are based on 3-4 digit SIC industry data, which suggests at least some degree of substitutability between the products of the different industries. Therefore, an organized industry's lobbying decisions may well be affected by the potential demand shift effects.

5 Substitutability and lobbying activity

So far, as in the original GH model, we assumed an exogenous lobby group structure. But why are only some industries organized? In the previous section, we demonstrated that if industry i is organized, the import tax for the substitute product $-i$ increases with the degree of substitutability. Hence, a sufficiently high degree of substitutability may entail a situation where a non-organized industry becomes protected without paying for it. That is, there may be free-riding in lobbying.

Let us illustrate this reasoning with an example. We continue to consider the 2-sector-2-firm model of the previous section but impose several additional assumptions. First, assume that the non-numeraire products are perfect substitutes, $\sigma = 1$. Second, assume that the industries have the same size $\alpha_1 = \alpha_2 = \alpha$. We allow some of the voters to own no claims to any of the industries' profit but the labor only, so $\alpha \leq 1/2$. To study the free-riding behavior, we extend the existing game by allowing for an initial stage 0 of costless lobby formation. In this stage, industries simultaneously decide whether they are getting organized. The industries deciding to form a lobby participate in the standard lobbying game in stage 1.

Using backward induction, we start by discussing stage 1. Compare the equilibrium outcomes under the two regimes: (i) when only industry 1 is organized and (ii) when both industries 1 and 2 are organized. Once more, we concentrate on interior solutions.

If only industry 1 is organized, the trade tariffs in this economy are determined by equations (20) and (21) for $\sigma = 1$. This entails the equilibrium tariffs

$$\widetilde{\tau}_1 (1, \alpha, a) = \widetilde{\tau}_2 (1, \alpha, a) \equiv \widetilde{\tau} = \frac{(1-c)}{4} \frac{5a + \alpha + 2}{(5a + 7\alpha - 1)}. \tag{22}$$

As mentioned above, the import taxes on both goods are the same, even though only one of the industries is organized. Under perfect substitutability, industry 1 treats the foreign producers of both good 1 and good 2 as similar Cournot competitors. The symmetry of the setting implies that the preferred trade policy of the organized lobby does not differ with respect to the foreign firms in sectors 1 and 2. Thus, the government also sets identical trade tariffs for these two goods.

In this equilibrium, the non-organized industry 2 gains more from trade protection than the organized (and contributing) industry 1. Indeed, as the industries are identical and so are their trade tariffs, the *gross* welfare levels of industries 1 and 2, that is, the welfare before the lobbying contributions are paid, coincide. Formally,

$$W_1(\widetilde{\boldsymbol{\tau}}) = W_2(\widetilde{\boldsymbol{\tau}}),$$

where $\widetilde{\boldsymbol{\tau}}$ denotes the tariff vector in this equilibrium, $\widetilde{\boldsymbol{\tau}} = (\widetilde{\tau}_1, \widetilde{\tau}_2)$. However, the trade tariffs in this equilibrium differ from the first best level, which implies that the organized industry makes a positive contribution to the government. Therefore, the *net* welfare level of industry 1 is below the net welfare level of industry 2:

$$V_1(\widetilde{\boldsymbol{\tau}}) = W_1(\widetilde{\boldsymbol{\tau}}) - C_1(\widetilde{\boldsymbol{\tau}}) < W_2(\widetilde{\boldsymbol{\tau}}) = V_2(\widetilde{\boldsymbol{\tau}}).$$

In this regime, industry 2 benefits from the trade policy while not paying for it.

Now, consider the case when the domestic producers of both good 1 and good 2 are organized. If they both participate in lobbying, solving the equation (10) for $\sigma = 1$ and $\alpha_L = 2\alpha \leq 1$ yields the equilibrium import tariffs

$$\widetilde{\widetilde{\tau}}_1 (1, \alpha, a) = \widetilde{\widetilde{\tau}}_2 (1, \alpha, a) \equiv \widetilde{\widetilde{\tau}} = \frac{(1-c)}{4} \frac{5a + 2\alpha + 4}{5a + 14\alpha - 2}, \tag{23}$$

(see the Appendix). Note that with two industries actively lobbying, the import tax is further from the first-best level. Formally,

$$\widetilde{\widetilde{\tau}} \geq \widetilde{\tau} \geq \tau_0 (\sigma = 1) = \frac{1-c}{4}.^{12} \tag{24}$$

Indeed, due to the perfect substitutability ($\sigma = 1$) and equal size ($\alpha_1 = \alpha_2 = \alpha$), industries 1 and 2 have exactly the same preferences. Hence, when they both actively participate in lobbying, the resulting policy is more biased towards the interest groups' preferred import tax.

We are interested in the trade-off between the costs and benefits of this bias for both industry 1, which is lobbying in either equilibrium, and for industry 2, which only contributes in the second equilibrium. In other words, we want to compare the lobbies' payoffs between these two equilibria. We consider the case when organized industries play globally truthful strategies, that is when the contributions of lobby i are (globally) equal to the excess of the lobby's welfare over a certain threshold B_j,

$$C_j(\widetilde{\boldsymbol{\tau}}, B_j) = \max\left[0, W_j(\widetilde{\boldsymbol{\tau}}) - B_j\right].$$

All truthful equilibria are also locally truthful, so in our argument we can rely on the results obtained in equations (22) and (23).

Note that the tariff given by equation (23) corresponds to the equilibrium when both organized industries actively participate in lobbying. In the original GH setting, the interests of different industries are opposed to each other so, if organized, each industry indeed prefers to buy the protection. However, in the presence of substitutes, this outcome is not necessarily unique. Indeed, if both industries are allowed to lobby, but each of them can get protected without paying for it, the game may admit equilibria where only one of two organized industries is active. Alternatively, there can be equilibria where both industries lobby but make different contributions. In what follows, we concentrate on symmetric truthful equilibria, which seems to be natural given the symmetry of the setting.

Denote the truthful equilibrium when only industry 1 is organized by E_1 and a symmetric equilibrium when both industries are organized by $E_{1\&2}$. We start by evaluating the amount of contributions and the government surplus in these two equilibria.

Lemma 6 *a) The government is equally well off in E_1 and $E_{1\&2}$. b) The total contributions to the government are greater in $E_{1\&2}$ than in E_1.*

Proof. To understand these results, we need to calculate truthful equilibrium contributions and net welfare levels. In a truthful equilibrium, each lobby j chooses a scalar anchor B_j so that the government would be just indifferent between choosing the equilibrium policy τ and the policy τ^{-j}, which is defined as the policy chosen by the government if the contributions of lobby j were zero:

$$\boldsymbol{\tau}^{-j} = \arg\max_{\boldsymbol{\tau}\in\mathbf{T}} \sum_{i\in L, i\neq j} C_i(\boldsymbol{\tau}, B_i) + aW(\boldsymbol{\tau}). \tag{25}$$

Then, the contribution of lobby j solves

$$\sum_{i\in L, i\neq j} C_i(\boldsymbol{\tau}^{-j}, B_i) + aW(\boldsymbol{\tau}^{-j}) = \sum_{i\in L} C_i(\boldsymbol{\tau}, B_i) + aW(\boldsymbol{\tau}). \tag{26}$$

As a consistency check, we should have each lobby j making no contribution at the tariff vector τ^{-j}

$$W_j(\boldsymbol{\tau}^{-j}) \leq B_j, \tag{27}$$

as otherwise it can increase its reservation utility B_j at no cost.

We are now ready to apply this procedure. First, we establish that in E_1, the government receives exactly the payoff it would get in the first-best equilibrium. Indeed, if industry 1 were to contribute zero, the government would get no contributions at all, and would thus maximize the gross social welfare

$$\widetilde{\boldsymbol{\tau}}^{-j} = \arg\max_{\boldsymbol{\tau}\in\mathbf{T}} aW(\boldsymbol{\tau}) = \boldsymbol{\tau}_0. \tag{28}$$

This observation and the government indifference condition (26) immediately imply that in such an equilibrium, the government gets exactly $G(\boldsymbol{\tau}_0)$.

Now, we show that the government receives exactly as much in $E_{1\&2}$. Indeed, by the construction of the equilibrium, condition (27) holds and industry 1 does not make any contribution at the tariff vector $\widetilde{\widetilde{\boldsymbol{\tau}}}^{-1}$:

$$C_1(\widetilde{\widetilde{\boldsymbol{\tau}}}^{-1}, B_1) = \max\left[0, W_1(\widetilde{\widetilde{\boldsymbol{\tau}}}^{-1}) - B_1\right] = 0. \tag{29}$$

As the goods are perfect substitutes and the two industries are exactly alike, their gross welfare is the same under policy $\widetilde{\widetilde{\boldsymbol{\tau}}}^{-1}$. Moreover, as the equilibrium $E_{1\&2}$ is symmetric, the anchors of two industries are the same ($B_1 = B_2$). Therefore, industry 2 does not contribute anything at the tariff vector $\widetilde{\widetilde{\boldsymbol{\tau}}}^{-1}$ either. Hence, if the contributions of lobby 1 were zero, the government would choose the free-trade policy

$$\widetilde{\widetilde{\boldsymbol{\tau}}}^{-1} = \arg\max_{\boldsymbol{\tau}\in\mathbf{T}} aW(\boldsymbol{\tau}) = \boldsymbol{\tau}^0. \tag{30}$$

The same result holds for trade policy $\widetilde{\widetilde{\boldsymbol{\tau}}}^{-2}$. Condition (26) immediately implies that in such an equilibrium, the government's payoff is the same as in the first-best equilibrium without any lobbying, which proves part a) of the Lemma.

Part b) follows from part a) and our observation (24). Indeed, when both industries are organized, the equilibrium tariffs are higher than in the case when only one industry is lobbying, and thus further from the first-best outcome. Thus, the government requires higher contributions to compensate for the larger social welfare loss. ■

Thus, we see that the equilibrium with two organized industries provides the lobbies with higher gross welfare, but requires additional lobbying contributions. However, these contributions are now paid by two lobbies, as compared to the equilibrium with a single organized industry. So lobby 1, which was active in both equilibria, benefits more (or loses less) than lobby 2, as it can now share the costs. Still, it is not clear whether either of them actually wins or loses in the equilibrium $E_{1\&2}$. This question is answered in the following proposition.

Proposition 7 *a) Lobby* 1 *has a higher net welfare in* $E_{1\&2}$ *than in* E_1 *for any admissible parameter values. b) Lobby* 2 *has a lower net welfare in* $E_{1\&2}$ *than in* E_1*, if and only if* $\alpha > 1/7$*.*

Proof. We start by calculating lobbying contributions to the government in equilibrium E_1. From (26) and (28), it follows that

$$C_1\left(\widetilde{\boldsymbol{\tau}},\widetilde{B}_1\right) = aW\left(\boldsymbol{\tau}^0\right) - aW(\widetilde{\boldsymbol{\tau}}).$$

Hence, the net welfare of lobby 1 is

$$V_1(\widetilde{\boldsymbol{\tau}}) \equiv \widetilde{B_1} = W_1(\widetilde{\boldsymbol{\tau}}) - C_1(\widetilde{\boldsymbol{\tau}},\widetilde{B_1}) = W_1(\widetilde{\boldsymbol{\tau}}) - aW(\boldsymbol{\tau}^0) + aW(\widetilde{\boldsymbol{\tau}}). \tag{31}$$

As industry 2 is not organized, its net welfare is equal to its gross welfare,

$$V_2(\widetilde{\boldsymbol{\tau}}) = W_2(\widetilde{\boldsymbol{\tau}}). \tag{32}$$

In equilibrium $E_{1\&2}$, conditions (26), (29) and (30) determine the aggregate contributions

$$C_1(\widetilde{\widetilde{\boldsymbol{\tau}}},\widetilde{\widetilde{B_1}}) + C_2(\widetilde{\widetilde{\boldsymbol{\tau}}},\widetilde{\widetilde{B_2}}) = aW(\boldsymbol{\tau}^0) - aW(\widetilde{\widetilde{\boldsymbol{\tau}}}).$$

As this equilibrium is symmetric, the anchors of both lobbies are the same and so is their gross welfare. Hence, their contributions are also equal and comprise half of the aggregate amount

$$C_1(\widetilde{\widetilde{\boldsymbol{\tau}}},\widetilde{\widetilde{B_1}}) = C_2(\widetilde{\widetilde{\boldsymbol{\tau}}},\widetilde{\widetilde{B_2}}) = \frac{1}{2}\left(aW(\boldsymbol{\tau}^0) - aW(\widetilde{\widetilde{\boldsymbol{\tau}}})\right).$$

The payoff of either lobby group is thus

$$V_i(\widetilde{\widetilde{\boldsymbol{\tau}}}) \equiv \widetilde{\widetilde{B_i}} = W_i(\widetilde{\widetilde{\boldsymbol{\tau}}}) - C_i(\widetilde{\widetilde{\boldsymbol{\tau}}},\widetilde{\widetilde{B_i}}) = W_i(\widetilde{\widetilde{\boldsymbol{\tau}}}) - \frac{1}{2}\left(aW(\boldsymbol{\tau}^0) - aW(\widetilde{\widetilde{\boldsymbol{\tau}}})\right), \quad i = 1, 2. \tag{33}$$

So in order to determine whether industry 1 gains from the equilibrium with two lobbies, payoffs (31) and (33) must be compared. It can be shown (see the Appendix) that

$$V_1(\widetilde{\widetilde{\boldsymbol{\tau}}}) - V_1(\widetilde{\boldsymbol{\tau}}) = \frac{9a}{20}\frac{(1-2\alpha)^2(1-c)^2}{(5a+7\alpha-1)(5a+14\alpha-2)} \geq 0. \tag{34}$$

Similarly, the welfare difference of industry 2 is given by payoffs (32) and (33). Simplifying the expression (see the Appendix), we get

$$V_2(\widetilde{\widetilde{\boldsymbol{\tau}}}) - V_2(\widetilde{\boldsymbol{\tau}}) = \frac{9a}{20} \frac{(1-2\alpha)^2 (1-c)^2}{(5a+14\alpha-2)(5a+7\alpha-1)^2} (1-7\alpha) \begin{cases} < 0, \alpha < 1/7; \\ \geq 0, \alpha \geq 1/7. \end{cases} \quad (35)$$

■

So as long as the size of the industry is sufficiently large, it loses from participating in lobbying. The intuition behind this result is as follows: if the industries are very concentrated ($\alpha = 0$), they highly benefit from an increase in protection as the loss in their members' consumer welfare resulting from higher prices is negligible. With decreasing ownership concentration (higher α), more and more consumers in the industry lose from the price increase which results in a decrease in protection rates and industry welfare. And it may be the case that the gain of the industry from increased protection (due to this industry participation in lobbying) is not high enough to cover the necessary contribution for this increase.

We characterized the outcome of the symmetric truthful equilibrium when both industries are organized, *given* its existence. The following lemma establishes that it indeed exists.

Lemma 8 *In the lobbying game with perfect substitutability ($\sigma = 1$) and industries of identical size, there exists a symmetric truthful equilibrium.*

Proof. See the Appendix. ■

Now, we turn to the lobby formation stage. In this stage, each industry decides whether it gets organized (O) and buys influence in the next stage of the game, or stays non-organized (N) and remains passive in the lobbying game. The original GH setup entails a single equilibrium of a type (O,O): getting organized is a dominant strategy in that game as different lobbies' interests are strictly opposed to each other. The same happens in our setting if the industry's ownership structure is very concentrated (low α). Then, again, getting organized is a dominant strategy and a single (O,O) equilibrium emerges.

However, if an industry has a dispersed ownership structure (higher α), it prefers to commit not to lobby as long as the substitute industry will be lobbying. That is, the lobby formation stage is a "chicken game", where each industry prefers to be organized when the other is not and vise versa. The payoffs of the game are

Ind.1 / Ind.2	organized	non-organized
organized	$(A, A,)$	(B, C)
non-organized	(C, B)	(D, D)

where

$$\begin{aligned} A &= V_i(\widetilde{\widetilde{\boldsymbol{\tau}}}) = W_i(\widetilde{\widetilde{\boldsymbol{\tau}}}) - \frac{1}{2}\left(aW(\boldsymbol{\tau}^0) - aW(\widetilde{\widetilde{\boldsymbol{\tau}}})\right), \\ B &= V_1(\widetilde{\boldsymbol{\tau}}) = W_1(\widetilde{\boldsymbol{\tau}}) - aW(\boldsymbol{\tau}^0) + aW(\widetilde{\boldsymbol{\tau}}), \\ C &= V_2(\widetilde{\boldsymbol{\tau}}) = W_i(\widetilde{\boldsymbol{\tau}}), \\ D &= W_i(\boldsymbol{\tau}^0). \end{aligned}$$

As we have shown, $C > A > B > D$. This game has two pure strategy Nash equilibria, (O,N) and (N,O), and one mixed strategy equilibrium. The outcome (O,O) is no longer an equilibrium of the game. In other words, in the presence of substitutability, fewer industries get organized, and lobbying becomes less intense. This discussion is summarized in the following corollary.

Corollary 9 *In the game extended by the participation decision stage, a dispersed ownership structure weakens the incentives of the industries to get organized which, in turn, leads to a less intensive lobbying process.*

Note that aggregate net welfare of industries 1 and 2 is larger in $E_{1\&2}$ than in E_1, as

$$V_1(\widetilde{\widetilde{\boldsymbol{\tau}}}) + V_2(\widetilde{\widetilde{\boldsymbol{\tau}}}) - (V_1(\widetilde{\boldsymbol{\tau}}) + V_2(\widetilde{\boldsymbol{\tau}})) = \frac{9}{4} \frac{a^2 (1-2\alpha)^2 (1-c)^2}{(5a+14\alpha-2)(5a+7\alpha-1)^2} > 0.$$

The free-riding problem in lobbying can thus be summarized as follows: product substitutability produces a positive inter-industry externality from protection which, in turn, leads to dilution of the incentives to organize smaller protection than the industries desire. By continuity, this result also extends to markets with high, but not perfect substitutability.

6 Conclusion

This paper studies the impact of demand linkages on the determination of trade policy, and the intensity of inter-industry lobbying competition. We find that product substitutability both weakens industries' incentives to get organized and their lobbying incentives when they are organized. The finding may explain why empirical investigations based on the "Protection for Sale" model have suggested that the government is almost exclusively concerned with welfare rather than contributions by lobbies.

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A Appendix

A.1 Derivation of equation (10)

We begin by simplifying equation (9):

$$(a+\alpha_L)\nabla\left[\sum_{k=1}^{m}\tau_k n_k^* q_k^*(\boldsymbol{\tau})-\sum_{k=1}^{m}p_k d_k(\mathbf{p}(\boldsymbol{\tau}))+\hat{U}(d_1(\mathbf{p}(\boldsymbol{\tau})),..,d_m(\mathbf{p}(\boldsymbol{\tau})))\right] \\ +\sum_{k=1}^{m}(I_k+a)\nabla\pi_k(\boldsymbol{\tau})=\mathbf{0}.$$

Start by evaluating

$$\frac{\partial}{\partial\tau_j}\left[\sum_{k=1}^{m}\tau_k n_k^* q_k^*(\boldsymbol{\tau})-\sum_{k=1}^{m}p_k d_k(\mathbf{p}(\boldsymbol{\tau}))+\hat{U}(d_1(\mathbf{p}(\boldsymbol{\tau})),...,d_m(\mathbf{p}(\boldsymbol{\tau})))\right]. \tag{36}$$

As

$$\frac{\partial}{\partial\tau_j}\left(\sum_{k=1}^{m}\tau_k n_k^* q_k^*(\boldsymbol{\tau})\right)=n_j^* q_j^*(\boldsymbol{\tau})+\sum_{k=1}^{m}\tau_k n_k^*\frac{\partial q_k^*(\boldsymbol{\tau})}{\partial\tau_j},$$

$$\frac{\partial}{\partial\tau_j}\left(-\sum_{k=1}^{m}p_k d_k(\mathbf{p}(\boldsymbol{\tau}))\right)=\left(-\sum_{k=1}^{m}\frac{\partial p_k}{\partial\tau_j}d_k(\mathbf{p}(\boldsymbol{\tau}))-\sum_{k=1}^{m}p_k\frac{\partial d_k(\mathbf{p}(\boldsymbol{\tau}))}{\partial\tau_j}\right),$$

and

$$\begin{aligned}\frac{\partial}{\partial\tau_j}\left(\hat{U}(d_1(\mathbf{p}(\boldsymbol{\tau})),..,d_m(\mathbf{p}(\boldsymbol{\tau})))\right) &= \sum_{i=1}^{m}\frac{\partial\hat{U}(d_1(\mathbf{p}(\boldsymbol{\tau})),..,d_m(\mathbf{p}(\boldsymbol{\tau})))}{\partial\left(d_i(\mathbf{p}(\boldsymbol{\tau}))\right)}*\frac{\partial d_i(\mathbf{p}(\boldsymbol{\tau}))}{\partial\tau_j}\\ &= \sum_{i=1}^{m}p_i\frac{\partial d_i(\mathbf{p}(\boldsymbol{\tau}))}{\partial\tau_j},\end{aligned}$$

expression (36) is equivalent to

$$\frac{\partial}{\partial\tau_j}\left[\sum_{k=1}^{m}\tau_k n_k^* q_k^*(\boldsymbol{\tau})-\sum_{k=1}^{m}p_k d_k(\mathbf{p}(\boldsymbol{\tau}))+\hat{U}(d_1(\mathbf{p}(\boldsymbol{\tau})),..,d_m(\mathbf{p}(\boldsymbol{\tau})))\right]= \\ n_j^* q_j^*(\boldsymbol{\tau})+\sum_{k=1}^{m}\tau_k n_k^*\frac{\partial q_k^*(\boldsymbol{\tau})}{\partial\tau_j}-\sum_{k=1}^{m}\frac{\partial p_k}{\partial\tau_j}d_k(\mathbf{p}(\boldsymbol{\tau}))-\sum_{k=1}^{m}p_k\frac{\partial d_k(\mathbf{p}(\boldsymbol{\tau}))}{\partial\tau_j}+ \\ \sum_{i=1}^{m}p_i\frac{\partial d_i(\mathbf{p}(\boldsymbol{\tau}))}{\partial\tau_j}=n_j^* q_j^*(\boldsymbol{\tau})+\sum_{k=1}^{m}\tau_k n_k^*\frac{\partial q_k^*(\boldsymbol{\tau})}{\partial\tau_j}-\sum_{k=1}^{m}\frac{\partial p_k}{\partial\tau_j}d_k(\mathbf{p}(\boldsymbol{\tau})).$$

Thus, each line of condition (9) takes the form

$$(a+\alpha_L)\left(n_j^* q_j^*(\boldsymbol{\tau})+\sum_{k=1}^{m}\tau_k n_k^*\frac{\partial q_k^*(\boldsymbol{\tau})}{\partial\tau_j}-\sum_{k=1}^{m}\frac{\partial p_k}{\partial\tau_j}d_k(\mathbf{p}(\boldsymbol{\tau}))\right)+\sum_{k=1}^{m}(I_k+a)n_k\frac{\partial\pi_k(\boldsymbol{\tau})}{\partial\tau_j}=\mathbf{0},$$

which can be rewritten as

$$-\sum_{k=1}^{m}\tau_k n_k^*\frac{\partial q_k^*(\boldsymbol{\tau})}{\partial\tau_j}=\sum_{k=1}^{m}\frac{(I_k+a)}{(a+\alpha_L)}n_k\frac{\partial\pi_k(\boldsymbol{\tau})}{\partial\tau_j}-\sum_{k=1}^{m}\frac{\partial p_k}{\partial\tau_j}d_k(\mathbf{p}(\boldsymbol{\tau}))+n_j^* q_j^*(\boldsymbol{\tau}). \tag{37}$$

Substituting definition $m_k = n_k^* q_k^*$ into expression (37) yields

$$-\sum_{k=1}^{m} \tau_k \frac{\partial m_k(\boldsymbol{\tau})}{\partial \tau_j} = \sum_{k=1}^{m} \frac{(I_k + a)}{(a + \alpha_L)} n_k \frac{\partial \pi_k(\boldsymbol{\tau})}{\partial \tau_j} - \sum_{k=1}^{m} \frac{\partial p_k}{\partial \tau_j} d_k(\mathbf{p}(\boldsymbol{\tau})) + n_j^* q_j^*(\boldsymbol{\tau}).$$

A.2 Proof of Lemma 1.

The first-order condition associated with the profit maximization problem of any of the n_k symmetric domestic firms in sector k (2) is given by

$$1 - n_k q_k - n_k^* q_k^* - \sigma \sum_{s \neq k} (n_s q_s + n_s^* q_s^*) = c_k + q_k. \tag{38}$$

Similarly, the first-order condition for any of the n_k^* symmetric foreign firms is given by

$$1 - n_k q_k - n_k^* q_k^* - \sigma \sum_{s \neq k} (n_s q_s + n_s^* q_s^*) = c_k + q_k^* + \tau_k. \tag{39}$$

It follows that

$$q_k = q_k^* + \tau_k \quad \forall k. \tag{40}$$

Let aggregate output in industry k be denoted

$$Q_k = n_k q_k + n_k^* q_k^* = N_k q_k - n_k^* \tau_k. \tag{41}$$

Then (38) for each k takes the form:

$$1 - \left(1 + \frac{1}{N_k}\right) Q_k - \sigma \sum_{l \neq k} Q_L = c_k - \frac{n_k^*}{N_k} \tau_k, \tag{42}$$

and the entire system of equations for all k transforms into

$$\begin{pmatrix} 1 + 1/N_1 & \sigma & \dots & \sigma \\ \sigma & 1 + 1/N_2 & \dots & \sigma \\ \dots & \dots & \dots & \dots \\ \sigma & \sigma & \dots & 1 + 1/N_m \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \\ \dots \\ Q_m \end{pmatrix} = \begin{pmatrix} 1 - c_1 - \frac{n_1^*}{N_1} \tau_1 \\ 1 - c_2 - \frac{n_2^*}{N_2} \tau_2 \\ \dots \\ 1 - c_m - \frac{n_m^*}{N_m} \tau_m \end{pmatrix}. \tag{43}$$

Let us study the matrix

$$\boldsymbol{S} = \begin{pmatrix} 1 + 1/N_1 & \sigma & \dots & \sigma \\ \sigma & 1 + 1/N_2 & \dots & \sigma \\ \dots & \dots & \dots & \dots \\ \sigma & \sigma & \dots & 1 + 1/N_m \end{pmatrix}.$$

First, the determinant of $\boldsymbol{S}$ is positive:

$$\det \boldsymbol{S} = \frac{N_1 + 1}{N_1} \prod_{i=2}^{m} \left(\frac{N_i + 1}{N_i} - \sigma \right) + \sigma \sum_{i=2}^{m} \left[\prod_{j=1, j \neq i}^{m} \left(\frac{N_j + 1}{N_j} - \sigma \right) \right] > 0.$$

Therefore, matrix $\boldsymbol{S}$ is invertible. Denote its inverse by $\boldsymbol{R} = \boldsymbol{S}^{-1}$. Then, the elements of matrix $\boldsymbol{R}$ can be written

$$R_{11} = \frac{1}{\det \boldsymbol{S}} \left(\frac{N_2 + 1}{N_2} \prod_{i=3}^{m} \left(\frac{N_i + 1}{N_i} - \sigma \right) + \sigma \sum_{i=3}^{m} \left[\prod_{j=2, j \neq i}^{m} \left(\frac{N_j + 1}{N_j} - \sigma \right) \right] \right) > 0,$$

$$R_{kk} = \frac{1}{\det \boldsymbol{S}}\left(\frac{N_1+1}{N_1}\prod_{i=2,i\neq k}^{m}\left(\frac{N_i+1}{N_i}-\sigma\right) + \sigma\sum_{i=2,i\neq k}^{m}\left[\prod_{j=1,j\neq i,j\neq k}^{m}\left(\frac{N_j+1}{N_j}-\sigma\right)\right]\right) > 0, \tag{44}$$

for $1 < k \leq m$, and

$$R_{ki} = \frac{1}{\det \boldsymbol{S}}\left(-\sigma\prod_{j=1,j\neq i,j\neq k}^{m}\left(\frac{N_j+1}{N_j}-\sigma\right)\right) \leq 0.$$

That is, matrix $\boldsymbol{R}$ is symmetric, its diagonal elements are positive, and the off-diagonal elements are non-positive. Therefore, system (43) can be rewritten as

$$\begin{pmatrix} Q_1 \\ Q_2 \\ \cdots \\ Q_m \end{pmatrix} = \boldsymbol{R}\begin{pmatrix} 1-c_1-\frac{n_1^*}{N_1}\tau_1 \\ 1-c_2-\frac{n_1^*}{N_2}\tau_1 \\ \cdots \\ 1-c_m-\frac{n_m^*}{N_m}\tau_m \end{pmatrix}, \tag{45}$$

and we see that

$$\frac{\partial Q_i}{\partial \tau_j}\begin{cases} < 0, i = j \\ > 0, i \neq j \end{cases}. \tag{46}$$

Moreover, from (41), it follows that

$$\begin{pmatrix} q_1 \\ q_2 \\ \cdots \\ q_m \end{pmatrix} = \begin{pmatrix} \left(Q_1 + n_1^*\tau_1\right)/N_1 \\ \left(Q_2 + n_2^*\tau_2\right)/N_2 \\ \cdots \\ \left(Q_m + n_m^*\tau_m\right)/N_m \end{pmatrix}. \tag{47}$$

So

$$\frac{\partial q_k}{\partial \tau_k} = \frac{1}{N_k}\left(\frac{\partial Q_k}{\partial \tau_k} + n_k^*\right) = \frac{1}{N_k}\left(-\frac{n_k^*}{N_k}R_{kk} + n_k^*\right). \tag{48}$$

Next we show that for any k

$$R_{kk} < N_k. \tag{49}$$

>From formula (44), inequality (49) is equivalent to

$$\left(\frac{N_1+1}{N_1}\prod_{i=2,i\neq k}^{m}\left(\frac{N_i+1}{N_i}-\sigma\right) + \sigma\sum_{i=2,i\neq k}^{m}\left[\prod_{j=1,j\neq i,j\neq k}^{m}\left(\frac{N_j+1}{N_j}-\sigma\right)\right]\right) - N_k \det \boldsymbol{S} < 0.$$

Denote this difference by D. It is equal to

$$
\begin{aligned}
D &= \left(\frac{N_1+1}{N_1}\prod_{i=2,i\neq k}^{m}\left(\frac{N_i+1}{N_i}-\sigma\right)+\sigma\sum_{i=2,i\neq k}^{m}\left[\prod_{\substack{j=1,\\ j\neq i,j\neq k}}^{m}\left(\frac{N_j+1}{N_j}-\sigma\right)\right]\right)-N_k\det\boldsymbol{S}\\
&= \frac{N_1+1}{N_1}\prod_{i=2,i\neq k}^{m}\left(\frac{N_i+1}{N_i}-\sigma\right)+\sigma\sum_{i=2,i\neq k}^{m}\left[\prod_{\substack{j=1,\\ j\neq i,j\neq k}}^{m}\left(\frac{N_j+1}{N_j}-\sigma\right)\right]\\
&\quad -N_k\left(\frac{N_1+1}{N_1}\prod_{i=2}^{m}\left(\frac{N_i+1}{N_i}-\sigma\right)+\sigma\sum_{i=2}^{m}\left[\prod_{j=1,j\neq i}^{m}\left(\frac{N_j+1}{N_j}-\sigma\right)\right]\right).
\end{aligned}
$$

By rearranging terms and simplifying, we rewrite it as

$$
\begin{aligned}
D &= N_k(\sigma-1)\left(\frac{N_1+1}{N_1}\prod_{i=2,i\neq k}^{m}\left(\frac{N_i+1}{N_i}-\sigma\right)+\sigma\sum_{i=2,i\neq k}^{m}\left[\prod_{\substack{j=1,\\ j\neq i,j\neq k}}^{m}\left(\frac{N_j+1}{N_j}-\sigma\right)\right]\right)\\
&\quad -\sigma\prod_{j=1,j\neq k}^{m}\left(\frac{N_j+1}{N_j}-\sigma\right).
\end{aligned}
$$

As $\sigma\in[0,1]$, $D<0$, proving that $R_{kk}<N_k$ as desired.

Applying this result to equality (48), we conclude that

$$\frac{\partial q_k}{\partial\tau_k}=\frac{1}{N_k}\left(-\frac{n_k^*}{N_k}R_{kk}+n_k^*\right)>\frac{n_k^*}{N_k}\left(-\frac{N_k}{N_k}+1\right)=0.$$

Equalities (46) and (47) imply that

$$\frac{\partial q_k}{\partial\tau_j}=\frac{1}{N_k}\frac{\partial Q_k}{\partial\tau_j}>0,\ \ k\neq j.$$

Finally, relations (40) and (41) suggest that

$$\begin{pmatrix} q_1^* \\ q_2^* \\ \dots \\ q_m^* \end{pmatrix}=\begin{pmatrix} (Q_1-n_1\tau_1)/N_1 \\ (Q_2-n_2\tau_2)/N_2 \\ \dots \\ (Q_m-n_m\tau_m)/N_m \end{pmatrix}.$$

As a result, by using inequalities (46), we conclude that

$$\frac{\partial q_k^*}{\partial\tau_k}=\frac{1}{N_k}\left(\frac{\partial Q^k}{\partial\tau_k}-n_k\right)<0$$

and

$$\frac{\partial q_k^*}{\partial\tau_j}=\frac{1}{N_k}\frac{\partial Q_k}{\partial\tau_j}>0,\ \ k\neq j.$$

A.3 Proof of Lemma 3.

Equality (40) allows us to rewrite the first-order conditions(39) as

$$1-(N_k+1)\,q_k^*-\sigma\sum_{s\neq k}N_s q_s^*=c_k+(n_k+1)\,\tau_k+\sigma\sum_{s\neq k}n_s\tau_s.$$

This can be rewritten in a matrix form as

$$\begin{pmatrix} 1+N_1 & \sigma N_2 & ... & \sigma N_m \\ \sigma N_1 & 1+N_2 & ... & \sigma N_m \\ ... & ... & ... & ... \\ \sigma N_1 & \sigma N_2 & ... & 1+N_m \end{pmatrix}\begin{pmatrix} q_1^* \\ q_2^* \\ ... \\ q_m^* \end{pmatrix} \tag{50}$$

$$=\begin{pmatrix} 1-c_1 \\ 1-c_2 \\ ... \\ 1-c_m \end{pmatrix}-\begin{pmatrix} n_1+1 & n_2\sigma & ... & n_m\sigma \\ n_1\sigma & n_2+1 & ... & n_m\sigma \\ ... & ... & ... & ... \\ n_1\sigma & n_2\sigma & ... & n_m+1 \end{pmatrix}\begin{pmatrix} \tau_1 \\ \tau_2 \\ ... \\ \tau_m \end{pmatrix}.$$

Note that

$$\begin{pmatrix} 1+N_1 & \sigma N_2 & ... & \sigma N_m \\ \sigma N_1 & 1+N_2 & ... & \sigma N_m \\ ... & ... & ... & ... \\ \sigma N_1 & \sigma N_2 & ... & 1+N_m \end{pmatrix}=\begin{pmatrix} \frac{1+N_1}{N_1} & \sigma & ... & \sigma \\ \sigma & \frac{1+N_2}{N_2} & ... & \sigma \\ ... & ... & ... & ... \\ \sigma & \sigma & ... & \frac{1+N_m}{N_m} \end{pmatrix}\begin{pmatrix} N_1 & 0 & ... & 0 \\ 0 & N_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & N_m \end{pmatrix}.$$

The first matrix in this product is matrix $\boldsymbol{S}$ defined above, and we have shown that it is invertible. Clearly, the diagonal matrix

$$\boldsymbol{N}=\begin{pmatrix} N_1 & 0 & ... & 0 \\ 0 & N_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & N_m \end{pmatrix}$$

is invertible as $N_i>0$ for all $i=1,...m$. Denote its inverse by $\boldsymbol{N}^{-1}$. Therefore, equation (50) can be rewritten as

$$\begin{pmatrix} q_1^* \\ q_2^* \\ ... \\ q_m^* \end{pmatrix}=\boldsymbol{N}^{-1}\boldsymbol{R}\left[\begin{pmatrix} 1-c_1 \\ 1-c_2 \\ ... \\ 1-c_m \end{pmatrix}-\begin{pmatrix} n_1+1 & n_2\sigma & ... & n_m\sigma \\ n_1\sigma & n_2+1 & ... & n_m\sigma \\ ... & ... & ... & ... \\ n_1\sigma & n_2\sigma & ... & n_m+1 \end{pmatrix}\begin{pmatrix} \tau_1 \\ \tau_2 \\ ... \\ \tau_m \end{pmatrix}\right].$$

It follows that the matrix of the derivatives of the output of a single foreign firm in sector k with respect to the trade tariffs in sector i is given by

$$\left(\frac{\partial q_k^*}{\partial \tau_j}\right)=-\boldsymbol{N}^{-1}\boldsymbol{R}\begin{pmatrix} n_1+1 & n_2\sigma & ... & n_m\sigma \\ n_1\sigma & n_2+1 & ... & n_m\sigma \\ ... & ... & ... & ... \\ n_1\sigma & n_2\sigma & ... & n_m+1 \end{pmatrix}.$$

As a result, the matrix of the derivatives of the aggregate import in sector k with respect to the trade tariffs is

$$\begin{aligned} M &= \begin{pmatrix} n_1^* & 0 & ... & 0 \\ 0 & n_2^* & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & n_m^* \end{pmatrix} \left(\frac{\partial q_k^*}{\partial \tau_j}\right) \\ &= -\begin{pmatrix} n_1^* & 0 & ... & 0 \\ 0 & n_2^* & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & n_m^* \end{pmatrix} \boldsymbol{N}^{-1}\boldsymbol{R} \begin{pmatrix} n_1+1 & n_2\sigma & ... & n_m\sigma \\ n_1\sigma & n_2+1 & ... & n_m\sigma \\ ... & ... & ... & ... \\ n_1\sigma & n_2\sigma & ... & n_m+1 \end{pmatrix}. \end{aligned}$$

Note that

$$\begin{pmatrix} n_1+1 & n_2\sigma & ... & n_m\sigma \\ n_1\sigma & n_2+1 & ... & n_m\sigma \\ ... & ... & ... & ... \\ n_1\sigma & n_2\sigma & ... & n_m+1 \end{pmatrix} = \begin{pmatrix} \frac{n_1+1}{n_1} & \sigma & ... & \sigma \\ \sigma & \frac{n_2+1}{n_2} & ... & \sigma \\ ... & ... & ... & ... \\ \sigma & \sigma & ... & \frac{n_m+1}{n_m} \end{pmatrix} \begin{pmatrix} n_1 & 0 & ... & 0 \\ 0 & n_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & n_m \end{pmatrix}.$$

Clearly, as $n_i > 0$, the inverse of

$$\begin{pmatrix} n_1 & 0 & ... & 0 \\ 0 & n_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & n_m \end{pmatrix}$$

is a diagonal matrix with positive elements. Similarly to our derivation in the proof of Lemma 1, the matrix

$$\boldsymbol{s} = \begin{pmatrix} \frac{n_1+1}{n_1} & \sigma & ... & \sigma \\ \sigma & \frac{n_2+1}{n_2} & ... & \sigma \\ ... & ... & ... & ... \\ \sigma & \sigma & ... & \frac{n_m+1}{n_m} \end{pmatrix}$$

is also invertible. Its determinant is positive and equal to

$$\det \boldsymbol{s} = \frac{n_1+1}{n_1}\prod_{i=2}^{m}\left(\frac{n_i+1}{n_i} - \sigma\right) + \sigma\sum_{i=2}^{m}\left[\prod_{j=1,j\neq i}^{m}\left(\frac{n_j+1}{n_j} - \sigma\right)\right] > 0.$$

Denote its inverse by $\boldsymbol{r} = \boldsymbol{s}^{-1}$. Then, the elements of matrix $\boldsymbol{r}$ are given by

$$r_{11} = \frac{1}{\det \boldsymbol{s}}\left(\frac{n_2+1}{n_2}\prod_{i=3}^{m}\left(\frac{n_i+1}{n_i} - \sigma\right) + \sigma\sum_{i=3}^{m}\left[\prod_{j=2,j\neq i}^{m}\left(\frac{n_j+1}{n_j} - \sigma\right)\right]\right) > 0,$$

$$r_{kk} = \frac{1}{\det \boldsymbol{s}}\left(\frac{n_1+1}{n_1}\prod_{i=2,i\neq k}^{m}\left(\frac{n_i+1}{n_i} - \sigma\right) + \sigma\sum_{i=2,i\neq k}^{m}\left[\prod_{j=1,j\neq i,j\neq k}^{m}\left(\frac{n_j+1}{n_j} - \sigma\right)\right]\right) > 0,$$

and

$$r_{ki} = \frac{1}{\det \boldsymbol{s}}\left(-\sigma\prod_{j=1,j\neq i,j\neq k}^{m}\left(\frac{n_j+1}{n_j} - \sigma\right)\right) \leq 0.$$

Therefore, matrix M is invertible as a product of invertible matrices. It remains to show that all elements of its inverse are non-positive, i.e., that

$$M^{-1} = \begin{pmatrix} n_1 & 0 & \dots & 0 \\ 0 & n_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & n_m \end{pmatrix}^{-1} \boldsymbol{rSN} \begin{pmatrix} n_1^* & 0 & \dots & 0 \\ 0 & n_2^* & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & n_m^* \end{pmatrix}^{-1} \geqq \mathbf{0}\,. \tag{51}$$

Lemma 10 *The matrix* $\boldsymbol{rSN} \geqq \mathbf{0}$.

Proof. As matrix $\boldsymbol{SN}$ is

$$\boldsymbol{SN} = \begin{pmatrix} 1+N_1 & \sigma N_2 & \dots & \sigma N_m \\ \sigma N_1 & 1+N_2 & \dots & \sigma N_m \\ \dots & \dots & \dots & \dots \\ \sigma N_1 & \sigma N_2 & \dots & 1+N_m \end{pmatrix},$$

the diagonal element (k,k) of the matrix $\boldsymbol{rSN}$ is given by

$$\left((1+N_k) r_{kk} + \sigma N_k \sum_{i=1, i\neq k}^{m} r_{ki} \right).$$

As the determinant of the matrix $\boldsymbol{s} = \boldsymbol{r}^{-1}$ is positive, the sign of this element does not change from multiplication by det $\boldsymbol{s}$. Therefore, the sign of the diagonal element (k,k) of the matrix $\boldsymbol{rSN}$ is equal to the sign of the following expression:

$$\begin{aligned}
& \det \boldsymbol{s} * \left((1+N_k) r_{kk} + \sigma N_k \sum_{i=1, i\neq k}^{m} r_{ki} \right) \\
= \ & (1+N_k) \left(\frac{n_1+1}{n_1} \prod_{i=2, i\neq k}^{m} \left(\frac{n_i+1}{n_i} - \sigma \right) + \sigma \sum_{i=2, i\neq k}^{m} \left[\prod_{j=1, j\neq i, j\neq k}^{m} \left(\frac{n_j+1}{n_j} - \sigma \right) \right] \right) \\
& -\sigma^2 N_k \sum_{i=1, i\neq k}^{m} \prod_{j=1, j\neq i, j\neq k}^{m} \left(\frac{n_j+1}{n_j} - \sigma \right) \\
> \ & (1+N_k) \left(\frac{n_1+1}{n_1} \prod_{i=2, i\neq k}^{m} \left(\frac{n_i+1}{n_i} - \sigma \right) \right) - \sigma^2 N_k \prod_{j=2,, j\neq k}^{m} \left(\frac{n_j+1}{n_j} - \sigma \right) \qquad (52) \\
= \ & \left((1+N_k) \frac{n_1+1}{n_1} - \sigma^2 N_k \right) \left(\frac{n_1+1}{n_1} \prod_{i=2, i\neq k}^{m} \left(\frac{n_i+1}{n_i} - \sigma \right) \right) > 0,
\end{aligned}$$

where the inequality in (52) follows from $\sigma > \sigma^2$, $1 + N_k > N_k$ and

$$\prod_{j=1, j\neq i, j\neq k}^{m} \left(\frac{n_j+1}{n_j} - \sigma \right) > 0.$$

Similarly, the sign of the off-diagonal element (k, j) is equal to the sign of the expression

$$\begin{aligned}
&\det \boldsymbol{s*}\left((1+N_j)r_{kj}+\sigma N_j\sum_{i=1,i\neq j,i\neq k}^{m} r_{ki}+\sigma N_j r_{kk}\right)\\
=\ & -(1+N_j)\,\sigma\prod_{i=1,i\neq j,i\neq k}^{m}\left(\frac{n_i+1}{n_i}-\sigma\right)-\sigma^2 N_j\sum_{i=1,i\neq k,i\neq j}^{m}\prod_{l=1,l\neq i,l\neq k}^{m}\left(\frac{n_l+1}{n_l}-\sigma\right)\\
&+\sigma N_j\left(\frac{n_1+1}{n_1}\prod_{i=2,i\neq k}^{m}\left(\frac{n_i+1}{n_i}-\sigma\right)+\sigma\sum_{i=2,i\neq k}^{m}\left[\prod_{l=1,l\neq i,l\neq k}^{m}\left(\frac{n_l+1}{n_l}-\sigma\right)\right]\right)\\
=\ & \left(-\sigma\,(1+N_j)\left(\frac{n_1+1}{n_1}-\sigma\right)-\sigma^2 N_j\left(\frac{n_j+1}{n_j}-\sigma\right)\right.\\
&\left.+\sigma N_j\left(\frac{n_1+1}{n_1}-\sigma\right)+N_j\left(\frac{n_1+1}{n_1}\right)\left(\frac{n_j+1}{n_j}-\sigma\right)\right)\left(\prod_{i=2,i\neq k,i\neq j}^{m}\left(\frac{n_i+1}{n_i}-\sigma\right)\right)\\
=\ & \sigma\left((N_j-n_j)\,\frac{n_1(1-\sigma)+1}{n_1 n_j}\right)\left(\prod_{i=2,i\neq k,i\neq j}^{m}\left(\frac{n_i+1}{n_i}-\sigma\right)\right)\geq 0
\end{aligned}$$

Therefore, all elements of matrix $\boldsymbol{rSN}$ are nonnegative. ■

Clearly, both

$$\begin{pmatrix} n_1 & 0 & ... & 0\\ 0 & n_2 & ... & 0\\ ... & ... & ... & ...\\ 0 & 0 & ... & n_m \end{pmatrix}^{-1}$$

and

$$\begin{pmatrix} n_1^* & 0 & ... & 0\\ 0 & n_2^* & ... & 0\\ ... & ... & ... & ...\\ 0 & 0 & ... & n_m^* \end{pmatrix}^{-1}$$

are positive diagonal matrices, so the product of these and a nonnegative matrix is once more non-negative. From Lemma 10, it immediately follows that matrix M^{-1} determined by equality (51) is non-positive.

A.4 Derivation of equation (19).

The output levels of the foreign and domestic firms in sectors 1 and 2 are given by equations (17) and (18). Using them, we obtain the following formulas for the tariff sensitivity of the imports

$$\frac{\partial m_k(\boldsymbol{\tau})}{\partial \tau_k}=\frac{-2\left(3-\sigma^2\right)}{9-4\sigma^2},$$

$$\frac{\partial m_k(\boldsymbol{\tau})}{\partial \tau_{-k}}=\frac{\sigma}{9-4\sigma^2};$$

domestic profits

$$\frac{\partial \pi_k(\boldsymbol{\tau})}{\partial \tau_k} = 2\left(3-2\sigma^2\right)\frac{(1-c)(3-2\sigma)+\left(3-2\sigma^2\right)\tau_k+\sigma\tau_{-k}}{\left(9-4\sigma^2\right)^2},$$

$$\frac{\partial \pi_k(\boldsymbol{\tau})}{\partial \tau_{-k}} = 2\sigma\frac{(1-c)(3-2\sigma)+\left(3-2\sigma^2\right)\tau_k+\sigma\tau_{-k}}{\left(9-4\sigma^2\right)^2};$$

price

$$\frac{\partial p_k(\boldsymbol{\tau})}{\partial \tau_k} = \frac{\left(3-2\sigma^2\right)}{9-4\sigma^2},$$

$$\frac{\partial p_k(\boldsymbol{\tau})}{\partial \tau_k} = \frac{\sigma}{9-4\sigma^2};$$

and aggregate domestic consumption in sectors 1 and 2

$$d_k(\mathbf{p}(\boldsymbol{\tau})) = \frac{2(3-2\sigma)(1-c)-3\tau_k+2\sigma\tau_{-k}}{9-4\sigma^2}.$$

Substituting these relations into the system (10) yields a linear system

$$\begin{aligned} & 2\left(3-\sigma^2\right)\tau_k-\sigma\tau_{-k} \\ = \; & \frac{(I_k+a)}{(a+\alpha_L)}2\left(3-2\sigma^2\right)\frac{(1-c)(3-2\sigma)+\left(3-2\sigma^2\right)\tau_k+\sigma\tau_{-k}}{\left(9-4\sigma^2\right)} \\ & +\frac{(I_{-k}+a)}{(a+\alpha_L)}2\sigma\frac{(1-c)(3-2\sigma)+\left(3-2\sigma^2\right)\tau_k+\sigma\tau_{-k}}{\left(9-4\sigma^2\right)} \\ & -\left(3-2\sigma^2\right)\frac{2(3-2\sigma)(1-c)-3\tau_k+2\sigma\tau_{-k}}{9-4\sigma^2} \\ & -\sigma\frac{2(3-2\sigma)(1-c)-3\tau_{-k}+2\sigma\tau_k}{9-4\sigma^2}+(1-c)(3-2\sigma)-2\left(3-\sigma^2\right)\tau_k+\sigma\tau_{-k}, \end{aligned}$$

$k=1,2$. Collecting terms and multiplying by a common factor, we get an equivalent system

$$\begin{aligned} & \left[-2\left(3-2\sigma^2\right)^2\frac{(I_k+a)}{(a+\alpha_L)}-2\sigma\left(3-2\sigma^2\right)\frac{(I_{-k}+a)}{(a+\alpha_L)}-76\sigma^2+16\sigma^4+99\right]\tau_k \\ & -\sigma\left[2\left(3-2\sigma^2\right)\frac{(a+I_k)}{(a+\alpha_L)}+2\sigma\frac{(a+I_{-k})}{(a+\alpha_L)}+15-4\sigma^2\right]\tau_{-k} \\ = \; & (1-c)(3-2\sigma)\left[3-2\sigma+2\sigma\frac{(a+I_{-k})}{(a+\alpha_L)}+2\left(3-2\sigma^2\right)\frac{(a+I_k)}{(a+\alpha_L)}\right], \end{aligned} \tag{53}$$

which determines trade tariffs τ_1 and τ_2.

In the absence of organized lobbies, $I_1 = I_2 = \alpha_L = 0$. Therefore, $(I_k+a)/(a+\alpha_L) = 1$ for $k = 1, 2$, and the system can be simplified into two symmetric equations

$$\left(-20\sigma-4\sigma^2+4\sigma^3+27\right)\tau_k-\sigma(7-4\sigma)\tau_{-k} = (1-c)(3-2\sigma)^2$$

for $k = 1, 2$. Solving this yields

$$\tau_1 = \tau_2 = \frac{(1-c)}{3+\sigma}.$$

A.5 Derivation of equations (20) and (21).

Formulas (20) and (21) follow from solving system (53) for $I_1 = 1$, $\alpha_L = \alpha$, and $I_2 = 0$.

A.6 Proof of Proposition 4.

To show that the ratio $\frac{\tau_1(\sigma,\alpha,a)}{\tau_1^0(\sigma)}$ decreases with σ for any (a, α), we proceed in six steps.

1. Describe the necessary condition for the interior solution for the tariff.
2. Compose the ratio $T(\sigma, \alpha, a) = \frac{\tau_1(\sigma,\alpha,a)}{\tau_1^0(\sigma)}$, represent it as a ratio of two polynomials $T(\sigma, \alpha, a) = \frac{N(\sigma,\alpha,a)}{D(\sigma,\alpha,a)}$ and show that both the numerator $N(\sigma, \alpha, a)$ and the denominator $D(\sigma, \alpha, a)$ decline in σ.
3. Show that $T(\sigma, \alpha, a)$ declines in σ on $[0, 1]$ iff the ratio of the derivatives of $N(\sigma, \alpha, a)$ and $D(\sigma, \alpha, a)$, $R(\sigma, \alpha, a) \equiv \frac{\partial N(\sigma,\alpha,a)}{\partial \sigma} \Big/ \frac{\partial D(\sigma,\alpha,a)}{\partial \sigma}$ is higher than $T(\sigma, \alpha, a)$ for all $\sigma \in [0, 1]$ and (a, α) delivering an interior solution.
4. Introduce an auxiliary linear function $A(\sigma, \alpha, a)$ and show that $R(\sigma, \alpha, a) \geq A(\sigma, \alpha, a)$ for any admissible (σ, α, a).
5. Show that $A(\sigma, \alpha, a) \geq T(\sigma, \alpha, a)$ for any admissible (σ, α, a).
6. From steps 3, 4 and 5, conclude that $R(\sigma, \alpha, a) \geq T(\sigma, \alpha, a)$.

A.6.1 Necessary condition for the interior solution for the tariff

The industry 1 tariff $\tau_1(\sigma, \alpha, a)$ is given by equation (20) and the first-best tariff (in the absence of any lobbying) $\tau_1^0(\sigma)$ is determined by equation (19). So the ratio of the lobbying tariff to the first-best is

$$\begin{aligned} T(\sigma, \alpha, a) &= \frac{\tau_1(\sigma, \alpha, a)}{\tau_1^0(\sigma)} = \\ & (3+\sigma)\left[4(a+1)(a+2\alpha)\sigma^3 + \left(4\alpha^2 - 12a\alpha - 8a - 12a^2 - 12\alpha\right)\sigma^2 \right. \\ & \left. - \left(10a + 14\alpha + 26a\alpha + 13\alpha^2 + 9a^2\right)\sigma + (3a+\alpha+2)(9a+11\alpha)\right] / \\ & \left[4\sigma^4(a+2\alpha-1)(a+2\alpha) + \sigma^2\left(14a + 22\alpha - 118a\alpha - 81\alpha^2 - 45a^2\right) \right. \\ & \left. + (9a+11\alpha-2)(9a+11\alpha)\right]. \end{aligned}$$

We are interested in an interior solution. A necessary condition is that the foreign firm produces a non-negative amount of the good:

$$q^*(0, \tau_1, \tau_2) = \frac{(1-c) - 2\tau_1}{3} \geq 0.$$

This condition is equivalent to

$$\begin{gathered} \tau_1(0, \alpha, a) \leq \frac{(1-c)}{2} \Leftrightarrow \\ \frac{3a+\alpha+2}{9a+11\alpha-2} \leq \frac{1}{2} \Leftrightarrow \\ a + 3\alpha \geq 2. \end{gathered} \tag{54}$$

We assume that the necessary condition for the interior solution (54) always holds.

A.6.2 Ratio $T(\sigma,\alpha,a) = N(\sigma,\alpha,a) /\ D(\sigma,\alpha,a)$.

Denote the numerator of $T(\sigma,\alpha,a)$ by

$$\begin{aligned} N(\sigma,\alpha,a) &= (3+\sigma)\left[4(a+1)(a+2\alpha)\sigma^3 + \left(4\alpha^2 - 12a\alpha - 8a - 12a^2 - 12\alpha\right)\sigma^2 \right. \\ &\quad \left. - \left(10a + 14\alpha + 26a\alpha + 13\alpha^2 + 9a^2\right)\sigma + (3a+\alpha+2)(9a+11\alpha)\right], \end{aligned}$$

and the denominator by

$$\begin{aligned} D(\sigma,\alpha,a) &= \left[4\sigma^4(a+2\alpha-1)(a+2\alpha) + \sigma^2\left(14a + 22\alpha - 118a\alpha - 81\alpha^2 - 45a^2\right)\right. \\ &\quad \left. + (9a+11\alpha-2)(9a+11\alpha)\right], \end{aligned}$$

so that

$$T(\sigma,\alpha,a) = \frac{N(\sigma,\alpha,a)}{D(\sigma,\alpha,a)}.$$

Note that both $N(\sigma,\alpha,a)$ and $D(\sigma,\alpha,a)$ are decreasing in σ for any a,α. Indeed

$$\begin{aligned} N(\sigma,\alpha,a) &= 3(3a+\alpha+2)(9a+11\alpha) - 4\sigma\left(3a + 5\alpha + 9a\alpha + 7\alpha^2\right) \\ &\quad -\sigma^2\left(34a + 50\alpha + 62a\alpha + \alpha^2 + 45a^2\right) + 4\sigma^3\left(a + 3\alpha + 3a\alpha + \alpha^2\right) \\ &\quad +4\sigma^4(a+1)(a+2\alpha). \end{aligned}$$

The first derivative of $N(\sigma,\alpha,a)$ with respect to σ is

$$\begin{aligned} \frac{\partial N(\sigma,\alpha,a)}{\partial\sigma} &= -4\left(3a + 5\alpha + 9a\alpha + 7\alpha^2\right) - 2\sigma\left(34a + 50\alpha + 62a\alpha + \alpha^2 + 45a^2\right) \\ &\quad +12\sigma^2\left(a + 3\alpha + 3a\alpha + \alpha^2\right) + 16\sigma^3(a+1)(a+2\alpha). \end{aligned}$$

The second derivative of $N(\sigma,\alpha,a)$ is

$$\begin{aligned} \frac{\partial^2 N(\sigma,\alpha,a)}{\partial\sigma^2} &= -2\left(34a + 50\alpha + 62a\alpha + \alpha^2 + 45a^2\right) + 24\sigma\left(a + 3\alpha + 3a\alpha + \alpha^2\right) \\ &\quad +48\sigma^2(a+1)(a+2\alpha). \end{aligned}$$

As $(a+1)(a+2\alpha) > 0$, it is a convex quadratic parabola with the global minimum at

$$\sigma = -\frac{a + 3\alpha + 3a\alpha + \alpha^2}{(a+1)(a+2\alpha)} < 0,$$

so that the second derivative of $N(\sigma,\alpha,a)$ increases on $\sigma \in [0,1]$. This derivative is negative at $\sigma = 0$

$$\frac{\partial^2 N(0,\alpha,a)}{\partial\sigma^2} = -2\left(34a + 50\alpha + 62a\alpha + \alpha^2 + 45a^2\right) < 0,$$

and can be of either sign at $\sigma = 1$

$$\frac{\partial^2 N(1,\alpha,a)}{\partial\sigma^2} = 2\left(2a + 34\alpha + 22a\alpha + 11\alpha^2 - 21a^2\right).$$

If it is negative at $\sigma = 1$,

$$\frac{\partial^2 N(1,\alpha,a)}{\partial\sigma^2} < 0,$$

it is also negative at the entire $\sigma \in [0.1]$, meaning that the first derivative of $N(\sigma, \alpha, a)$ decreases in $\sigma \in [0, 1]$. If

$$\frac{\partial^2 N(1, \alpha, a)}{\partial \sigma^2} > 0,$$

it changes sign only once, so the first derivative of $N(1, \alpha, a)$ with respect to σ, $\frac{\partial N(1,\alpha,a)}{\partial \sigma}$, first declines and then increases on $\sigma \in [0, 1]$. As

$$\frac{\partial N(0, \alpha, a)}{\partial \sigma} = -4\left(3a + 5\alpha + 9a\alpha + 7\alpha^2\right) < 0$$

and

$$\frac{\partial N(1, \alpha, a)}{\partial \sigma} = -2(37a + 9\alpha + 26)(a + \alpha) < 0,$$

we can conclude that $\frac{\partial N(\sigma,\alpha,a)}{\partial \sigma} < 0$ for any $\sigma \in [0, 1]$, and, thus, that $N(\sigma, \alpha, a)$ is decreasing in σ.

Similarly,

$$\begin{aligned} D(\sigma, \alpha, a) &= (9a + 11\alpha - 2)(9a + 11\alpha) + \\ &\quad \sigma^2\left(14a + 22\alpha - 118a\alpha - 81\alpha^2 - 45a^2\right) + 4\sigma^4(a + 2\alpha - 1)(a + 2\alpha). \end{aligned}$$

The first derivative of $D(\sigma, \alpha, a)$ is

$$\frac{\partial D(\sigma, \alpha, a)}{\partial \sigma} = 2\sigma\left(14a + 22\alpha - 118a\alpha - 81\alpha^2 - 45a^2\right) + 16\sigma^3(a + 2\alpha - 1)(a + 2\alpha).$$

The second derivative of $D(\sigma, \alpha, a)$ is

$$\frac{\partial^2 D(\sigma, \alpha, a)}{\partial \sigma^2} = 2\left(14a + 22\alpha - 118a\alpha - 81\alpha^2 - 45a^2\right) + 48\sigma^2(a + 2\alpha - 1)(a + 2\alpha).$$

It is once more a convex quadratic parabola with the global minimum at $\sigma = 0$, so it increases on $\sigma \in [0, 1]$. As it is negative at $\sigma = 1$

$$\frac{\partial^2 D(1, \alpha, a)}{\partial \sigma^2} = -2\left(10a + 26\alpha + 22a\alpha - 15\alpha^2 + 21a^2\right) < 0,$$

it is thus negative at the entire segment $\sigma \in [0, 1]$. As a result, the first derivative $\frac{\partial D(\sigma,\alpha,a)}{\partial \sigma}$ decreases over σ. As

$$\frac{\partial D(0, \alpha, a)}{\partial \sigma} = 0,$$

we conclude that for any $\sigma \in [0, 1]$,

$$\frac{\partial D(\sigma, \alpha, a)}{\partial \sigma} \leq 0,$$

and thus, $D(\sigma, \alpha, a)$ is decreasing in σ.

A.6.3 Necessary and sufficient condition for $T(\sigma,\alpha,a)$ to decline in σ

The derivative of $T(\sigma,\alpha,a)$ is

$$\frac{\partial T(\sigma,\alpha,a)}{\partial\sigma}=\frac{\frac{\partial N(\sigma,\alpha,a)}{\partial\sigma}D(\sigma,\alpha,a)-\frac{\partial D(\sigma,\alpha,a)}{\partial\sigma}N(\sigma,\alpha,a)}{D^2(\sigma,\alpha,a)}.$$

It is negative if and only iff

$$\frac{\partial N(\sigma,\alpha,a)}{\partial\sigma}D(\sigma,\alpha,a)<\frac{\partial D(\sigma,\alpha,a)}{\partial\sigma}N(\sigma,\alpha,a). \tag{55}$$

As both $\frac{\partial N(\sigma,\alpha,a)}{\partial\sigma}$ and $\frac{\partial D(\sigma,\alpha,a)}{\partial\sigma}$ are negative, inequality (55) is equivalent to

$$\frac{\partial N(\sigma,\alpha,a)}{\partial\sigma}\Big/\frac{\partial D(\sigma,\alpha,a)}{\partial\sigma}>\frac{N(\sigma,\alpha,a)}{D(\sigma,\alpha,a)}\equiv T(\sigma,\alpha,a). \tag{56}$$

Denote

$$\begin{aligned}R(\sigma,\alpha,a)\equiv{}&\frac{\partial N(\sigma,\alpha,a)}{\partial\sigma}\Big/\frac{\partial D(\sigma,\alpha,a)}{\partial\sigma}=\\&\left(-4\left(3a+5\alpha+9a\alpha+7\alpha^2\right)-2\sigma\left(34a+50\alpha+62a\alpha+\alpha^2+45a^2\right)+\right.\\&\left.12\sigma^2\left(a+3\alpha+3a\alpha+\alpha^2\right)+16\sigma^3(a+1)(a+2\alpha)\right)/\\&\left(2\sigma\left(14a+22\alpha-118a\alpha-81\alpha^2-45a^2\right)+16\sigma^3(a+2\alpha-1)(a+2\alpha)\right).\end{aligned}$$

Substituting this notation into inequality (56), we see that $T(\sigma,\alpha,a)$ declines if and only if

$$R(\sigma,\alpha,a)>T(\sigma,\alpha,a).$$

A.6.4 Auxiliary function $A(\sigma,\alpha,a)$, such that $R(\sigma,\alpha,a)\geq A(\sigma,\alpha,a)$.

Define a linear function of σ $A(a,\alpha,\sigma)$,

$$A(a,\alpha,\sigma)=3\frac{3a+\alpha+2}{9a+11\alpha-2}-8\frac{a+\alpha+8a\alpha+6\alpha^2+2}{(9a+11\alpha-2)(37a+49\alpha-6)}\sigma.$$

Let us show that for any $\sigma\in[0,1]$

$$R(\sigma,\alpha,a)>A(a,\alpha,\sigma).$$

Consider

$$\begin{aligned}\Delta_{RA}={}&R(\sigma,\alpha,a)-A(a,\alpha,\sigma)\\={}&\frac{2(\sigma-1)}{\sigma(9a+11\alpha-2)(37a+49\alpha-6)}*\\&*[\left(14a+22\alpha-118a\alpha-81\alpha^2-45a^2\right)\\&+8\sigma^2(a+2\alpha-1)(a+2\alpha)]^{-1}\\&*\{\left(3a+5\alpha+9a\alpha+7\alpha^2\right)(37a+49\alpha-6)(9a+11\alpha-2)\\&+\sigma(37a+49\alpha-6)(2a+6\alpha+176a\alpha+105\alpha^2\\&-39\alpha^3-34a\alpha^2+9a^2\alpha+63a^2)\\&+8\sigma^2(a+2\alpha)\left(21a+3\alpha-17a\alpha-9\alpha^2+10\right)(9a+11\alpha-2)\\&+32\sigma^3(a+2\alpha)(a+2\alpha-1)\left(a+\alpha+8a\alpha+6\alpha^2+2\right)\}\end{aligned} \tag{57}$$

Let us look at the signs of components of the product in (57). Note that

$$\frac{2(\sigma-1)}{\sigma\left((14a+22\alpha-118a\alpha-81\alpha^2-45a^2)+8\sigma^2(a+2\alpha-1)(a+2\alpha)\right)}=\frac{4(\sigma-1)}{D'_\sigma(\sigma,\alpha,a)}\geq 0.$$

The product $(9a+11\alpha-2)(37a+49\alpha-6)$ is positive as each of the factors is positive (a and α are nonnegative and we employ condition (54)):

$$9a+11\alpha-2=8(a+\alpha)+(a+3\alpha-2)>0,$$

$$37a+49\alpha-6=34a+40\alpha+3(a+3\alpha-2)>0.$$

Similarly, the coefficients of the cubic polynomial with respect to σ in the numerator are all positive: the constant term

$$\left(3a+5\alpha+9a\alpha+7\alpha^2\right)(37a+49\alpha-6)(9a+11\alpha-2)>0.$$

The coefficient by σ is positive,

$$(37a+49\alpha-6)(2a+6\alpha+176a\alpha+105\alpha^2-39\alpha^3-34a\alpha^2+9a^2\alpha+63a^2)>0$$

as

$$\begin{aligned} 2a+6\alpha+176a\alpha+105\alpha^2-39\alpha^3-34a\alpha^2+9a^2\alpha+63a^2 &\geq \\ 2a+6\alpha+176a\alpha+105\alpha^2-39\alpha^2-34a\alpha+9a^2\alpha+63a^2 &= \\ a+6\alpha+142a\alpha+66\alpha^2+9a^2\alpha+63a^2 &> 0. \end{aligned}$$

The coefficient by σ^2 is also positive,

$$8(a+2\alpha)\left(21a+3\alpha-17a\alpha-9\alpha^2+10\right)(9a+11\alpha-2)>0,$$

as

$$21a+3\alpha-17a\alpha-9\alpha^2+10>21a+3\alpha-17a-9+10=4a+3\alpha+1>0.$$

And, finally, the coefficient by σ^3 is positive,

$$32\sigma^3(a+2\alpha)(a+2\alpha-1)\left(a+\alpha+8a\alpha+6\alpha^2+2\right)>0,$$

as

$$a+2\alpha-1>a+3\alpha-2\geq 0.$$

Thus, we can conclude that for any $\sigma\in[0,1]$

$$\Delta_{RA}=R(\sigma,\alpha,a)-A(a,\alpha,\sigma)\geq 0. \tag{58}$$

A.6.5 Proof of $A(\sigma,\alpha,a) \geq T(\sigma,\alpha,a)$

Now we show that

$$A(a,\alpha,\sigma) \geq T(\sigma,\alpha,a).$$

Consider the difference Δ_{TA} between $T(\sigma,\alpha,a)$ and $A(a,\alpha,\sigma)$:

$$\begin{aligned}
\Delta_{TA} &= T(\sigma,\alpha,a) - A(a,\alpha,\sigma) \\
&= \frac{4\sigma}{(9a+11\alpha-2)(37a+49\alpha-6)} / [(9a+11\alpha-2)(9a+11\alpha) \\
&\quad +\sigma^2\left(14a+22\alpha-118a\alpha-81\alpha^2-45a^2\right)+4\sigma^4(a+2\alpha-1)(a+2\alpha)] \\
&\quad *((9a+11\alpha-2)(54a+74\alpha-238a\alpha-181\alpha^2 \\
&\quad -211\alpha^3-416a\alpha^2-189a^2\alpha-93a^2)+2\sigma(37a+49\alpha-6) \qquad (59) \\
&\quad *\left(-2a-4\alpha-29a\alpha-16\alpha^2+29\alpha^3+49a\alpha^2+18a^2\alpha-9a^2\right) \\
&\quad +\sigma^2(68a+124\alpha+449a^2\alpha^2-912a\alpha-760\alpha^2+1555\alpha^3-433\alpha^4 \\
&\quad +2585a\alpha^2+1361a^2\alpha-247a\alpha^3+279a^3\alpha-280a^2+243a^3) \\
&\quad +2\sigma^3(1-\alpha)(37a+49\alpha-6)(5a+3\alpha+2)(a+2\alpha) \\
&\quad +8\sigma^4(a+2\alpha-1)\left(a+\alpha+8a\alpha+6\alpha^2+2\right)(a+2\alpha)).
\end{aligned}$$

Once more, we determine the signs of the components of the product in (59). We already know that

$$(9a+11\alpha-2)(37a+49\alpha-6) > 0.$$

Moreover,

$$\begin{aligned}
&4\sigma/[(9a+11\alpha-2)(9a+11\alpha)+\sigma^2\left(14a+22\alpha-118a\alpha-81\alpha^2-45a^2\right) \\
&+4\sigma^4(a+2\alpha-1)(a+2\alpha)] = \frac{4\sigma}{D(\sigma,\alpha,a)} \geq 0.
\end{aligned}$$

Consider the remaining component of the product. Denote it by

$$\begin{aligned}
M(\sigma,\alpha,a) = {} & (9a+11\alpha-2)(54a+74\alpha-238a\alpha-181\alpha^2 \\
& -211\alpha^3-416a\alpha^2-189a^2\alpha-93a^2)+2\sigma(37a+49\alpha-6) \\
& *\left(-2a-4\alpha-29a\alpha-16\alpha^2+29\alpha^3+49a\alpha^2+18a^2\alpha-9a^2\right) \\
& +\sigma^2(68a+124\alpha+449a^2\alpha^2-912a\alpha-760\alpha^2+1555\alpha^3-433\alpha^4 \\
& +2585a\alpha^2+1361a^2\alpha-247a\alpha^3+279a^3\alpha-280a^2+243a^3) \\
& +2\sigma^3(1-\alpha)(37a+49\alpha-6)(5a+3\alpha+2)(a+2\alpha) \\
& +8\sigma^4(a+2\alpha-1)\left(a+\alpha+8a\alpha+6\alpha^2+2\right)(a+2\alpha).
\end{aligned}$$

Its second derivative with respect to σ is a quadratic parabola,

$$\begin{aligned}
\frac{\partial^2 M(\sigma,\alpha,a)}{\partial\sigma^2} = {} & 2(68a+124\alpha+449a^2\alpha^2-912a\alpha-760\alpha^2+1555\alpha^3-433\alpha^4+ \\
& 2585a\alpha^2+1361a^2\alpha-247a\alpha^3+279a^3\alpha-280a^2+243a^3)+ \\
& 12\sigma(1-\alpha)(37a+49\alpha-6)(5a+3\alpha+2)(a+2\alpha)+ \\
& 96\sigma^2(a+2\alpha-1)\left(a+\alpha+8a\alpha+6\alpha^2+2\right)(a+2\alpha).
\end{aligned}$$

As $(a+2\alpha-1)\left(a+\alpha+8a\alpha+6\alpha^2+2\right)(a+2\alpha)>0$, this parabola is convex and has its minimum at $\sigma_{\min}=-\frac{(1-\alpha)(37a+49\alpha-6)(5a+3\alpha+2)}{16(a+2\alpha-1)(a+\alpha+8a\alpha+6\alpha^2+2)}<0$. Thus, it is increasing at $\sigma\in[0,1]$. It is positive at $\sigma=0$,

$$\begin{aligned}\frac{\partial^2 M(0,\alpha,a)}{\partial\sigma^2} &= 2(68a+124\alpha+449a^2\alpha^2-912a\alpha-760\alpha^2+1555\alpha^3-433\alpha^4\\&\quad+2585a\alpha^2+1361a^2\alpha-247a\alpha^3+279a^3\alpha-280a^2+243a^3)\\&= 2[(140a^2+456a\alpha+380\alpha^2)(a+3\alpha-2)+68a+124\alpha+449a^2\alpha^2\\&\quad+415\alpha^3-433\alpha^4+837a\alpha^2+485a^2\alpha-247a\alpha^3+279a^3\alpha+103a^3]\\&> 2\left(124\alpha^4+415\alpha^4-433\alpha^4+837a\alpha^2-247a\alpha^2\right)\\&= 4\alpha^2\left(295a+53\alpha^2\right)>0,\end{aligned}$$

and increasing in σ. So we conclude that for any $\sigma\in[0,1]$,

$$\frac{\partial^2 M(\sigma,\alpha,a)}{\partial\sigma^2}>0.$$

As a result, $M(\sigma,\alpha,a)$ is a convex function of σ at $[0,1]$ for any (α,a) and reaches its maximum at (either of) the corner points of the segment.

But $M(\sigma,\alpha,a)$ is negative both at $\sigma=0$ and $\sigma=1$. Indeed,

$$\begin{aligned}M(0,\alpha,a)=(9a+11\alpha-2)\,&[54a+74\alpha-238a\alpha\\&-181\alpha^2-211\alpha^3-416a\alpha^2-189a^2\alpha-93a^2]<0,\end{aligned}$$

as

$$\begin{aligned}&54a+74\alpha-238a\alpha-181\alpha^2-211\alpha^3-416a\alpha^2-189a^2\alpha-93a^2=\\&(27a+37\alpha)(2-3\alpha-a)-\left(120a\alpha+70\alpha^2+211\alpha^3+416a\alpha^2+189a^2\alpha+66a^2\right)<0.\end{aligned}$$

Moreover,

$$M(1,\alpha,a)=-2(a+\alpha)(9a+11\alpha-2)\left(49a+81\alpha+22a\alpha+14\alpha^2-14\right)<0,$$

as

$$49a+81\alpha+22a\alpha+14\alpha^2-14=7(a+3\alpha-2)+42a+60\alpha+22a\alpha+14\alpha^2>0.$$

So we conclude that

$$M(\sigma,\alpha,a)\le 0,$$

which is equivalent to

$$\Delta_{TA}=T(\sigma,\alpha,a)-A(a,\alpha,\sigma)\le 0. \tag{61}$$

A.6.6 Conclusion: $R(\sigma,\alpha,a)\ge A(\sigma,\alpha,a)\ge T(\sigma,\alpha,a)$

>From (58) and (61), it follows that

$$R(\sigma,\alpha,a)\ge T(\sigma,\alpha,a).$$

which implies that $\frac{\tau_1(\sigma,\alpha,a)}{\tau_1^0(\sigma)}$ decreases with σ for any admissible (a,α).

The result that the ratio $\frac{\tau_2(\sigma,\alpha,a)}{\tau_2^0(\sigma)}$ increases with σ for any (a,α) is proven in a similar way.

A.7 Derivation of equation (23)

Equation (23) results from solving system (53) for $I_1 = I_2 = 1$, $\sigma = 1$and $\alpha_L = 2\alpha$. For these parameters, the system consists of two symmetric equations

$$\left[-4\frac{(1+a)}{(a+2\alpha)}+39\right]\tau_k-\left[4\frac{(1+a)}{(a+2\alpha)}+11\right]\tau_{-k}=(1-c)\left[1+4\frac{(1+a)}{(a+2\alpha)}\right],$$

which yields

$$\widetilde{\widetilde{\tau}}_1(1,\alpha,a)=\widetilde{\widetilde{\tau}}_2(1,\alpha,a)=\frac{(1-c)}{4}\frac{5a+2\alpha+4}{5a+14\alpha-2}.$$

A.8 Derivation of equations (34) and (35)

>From equations (31) and (33), it follows that

$$\begin{aligned} V_1(\widetilde{\widetilde{\boldsymbol{\tau}}})-V_1(\widetilde{\boldsymbol{\tau}}) &= W_1(\widetilde{\widetilde{\boldsymbol{\tau}}})-\frac{1}{2}\left(aW(\boldsymbol{\tau}^0)-aW(\widetilde{\widetilde{\boldsymbol{\tau}}})\right)-\left(W_1(\widetilde{\boldsymbol{\tau}})-aW(\boldsymbol{\tau}^0)+aW(\widetilde{\boldsymbol{\tau}})\right) \\ &= W_1(\widetilde{\widetilde{\boldsymbol{\tau}}})-W_1(\widetilde{\boldsymbol{\tau}})+\frac{1}{2}a\left(W(\boldsymbol{\tau}^0)+W(\widetilde{\widetilde{\boldsymbol{\tau}}})-2W(\widetilde{\boldsymbol{\tau}})\right). \end{aligned} \tag{62}$$

Substituting the expressions for the outputs, profits, volume of imports and domestic consumption into the formulas for the lobby $i=1,2$, welfare (6) and aggregate social welfare (7), and simplifying the resulting expressions yields

$$W_i(\widetilde{\widetilde{\boldsymbol{\tau}}})=\frac{9}{4}(1-c)^2(a+2\alpha)\frac{a-2\alpha+14\alpha^2+3a\alpha}{(5a+14\alpha-2)^2},$$

$$W_i(\widetilde{\boldsymbol{\tau}})=\frac{9}{4}(1-c)^2(a+\alpha)\frac{a-\alpha+7\alpha^2+3a\alpha}{(5a+7\alpha-1)^2},$$

$$W(\boldsymbol{\tau}^0)=\frac{9}{20}(1-c)^2,$$

$$W(\widetilde{\boldsymbol{\tau}})=\frac{9}{4}(1-c)^2(a+\alpha)\frac{5a+9\alpha-2}{(5a+7\alpha-1)^2},$$

$$W(\widetilde{\widetilde{\boldsymbol{\tau}}})=\frac{9}{4}(1-c)^2(a+2\alpha)\frac{5a+18\alpha-4}{(5a+14\alpha-2)^2}.$$

Inserting into (62), we obtain

$$V_1(\widetilde{\widetilde{\boldsymbol{\tau}}})-V_1(\widetilde{\boldsymbol{\tau}})=\frac{9}{20}\frac{(1-c)^2a(1-2\alpha)^2}{(5a+7\alpha-1)(5a+14\alpha-2)}.$$

Similarly, equations (32), (33) and the formulas above imply that

$$\begin{aligned} V_2(\widetilde{\widetilde{\boldsymbol{\tau}}})-V_2(\widetilde{\boldsymbol{\tau}}) &= W_1(\widetilde{\widetilde{\boldsymbol{\tau}}})-\frac{1}{2}\left(aW(\boldsymbol{\tau}^0)-aW(\widetilde{\widetilde{\boldsymbol{\tau}}})\right)-W_2(\widetilde{\boldsymbol{\tau}}) \\ &= \frac{9}{20}\frac{(1-c)^2a(1-7\alpha)(1-2\alpha)^2}{(5a+14\alpha-2)(5a+7\alpha-1)^2}. \end{aligned}$$

A.9 Proof of Lemma (8)

If industry 2 chooses $\widetilde{\widetilde{B_2}}$ defined by expression (33), industry 1's best response is precisely setting $\widetilde{\widetilde{B_1}} = \widetilde{\widetilde{B_2}}$. Clearly, it is not in industry 1's interest to decrease $\widetilde{\widetilde{B_1}}$ (and hence its payoff, in case $\widetilde{\widetilde{B_1}}$ does not bind). We need to consider two cases. Assume first that $\alpha > 1/7$ so that

$$\widetilde{\widetilde{B_i}} = W_i(\tilde{\tilde{\boldsymbol{\tau}}}) - \frac{1}{2}\left(aW(\boldsymbol{\tau}^0) - aW(\tilde{\tilde{\boldsymbol{\tau}}})\right) < W_1(\tilde{\boldsymbol{\tau}}),$$

which implies that the government gets positive contributions both at policies $\tilde{\tilde{\boldsymbol{\tau}}}$ and $\tilde{\boldsymbol{\tau}}$. If industry 1 chooses $\widetilde{\widetilde{B_1}} > \widetilde{\widetilde{B_2}}$, the government prefers tariff $\boldsymbol{\tau}^0$ to $\tilde{\tilde{\boldsymbol{\tau}}}$ and $\tilde{\boldsymbol{\tau}}$. Indeed, by equation (8) $\tilde{\tilde{\boldsymbol{\tau}}}$ maximizes the sum of social welfare and the lobbies' welfare, so that the government prefers $\tilde{\tilde{\boldsymbol{\tau}}}$ to $\tilde{\boldsymbol{\tau}}$:

$$\begin{aligned} G(\tilde{\boldsymbol{\tau}}) &= aW(\tilde{\boldsymbol{\tau}}) + \left(W_1(\tilde{\boldsymbol{\tau}}) - \widetilde{\widetilde{B_1}}\right) + \left(W_2(\tilde{\boldsymbol{\tau}}) - \widetilde{\widetilde{B_2}}\right) \\ &< aW(\tilde{\tilde{\boldsymbol{\tau}}}) + \left(W_1(\tilde{\tilde{\boldsymbol{\tau}}}) - \widetilde{\widetilde{B_1}}\right) + \left(W_2(\tilde{\tilde{\boldsymbol{\tau}}}) - \widetilde{\widetilde{B_2}}\right) = G(\tilde{\tilde{\boldsymbol{\tau}}}). \end{aligned}$$

In turn, due to our assumption $\widetilde{\widetilde{B_1}} > \widetilde{\widetilde{B_2}}$, policy $\tilde{\tilde{\boldsymbol{\tau}}}$ does not pay the government enough to deviate from the first-best policy:

$$\begin{aligned} G(\tilde{\tilde{\boldsymbol{\tau}}}) &= aW(\tilde{\tilde{\boldsymbol{\tau}}}) + \left(W_1(\tilde{\tilde{\boldsymbol{\tau}}}) - \widetilde{\widetilde{B_1}}\right) + \frac{1}{2}\left(aW(\boldsymbol{\tau}^0) - aW(\tilde{\tilde{\boldsymbol{\tau}}})\right) \\ &= \left(W_1(\tilde{\tilde{\boldsymbol{\tau}}}) - \widetilde{\widetilde{B_1}}\right) + \frac{1}{2}\left(aW(\boldsymbol{\tau}^0) + aW(\tilde{\tilde{\boldsymbol{\tau}}})\right) < aW(\boldsymbol{\tau}^0). \end{aligned}$$

If instead $\alpha \le 1/7$, implying that

$$\widetilde{\widetilde{B_i}} = W_i(\tilde{\tilde{\boldsymbol{\tau}}}) - \frac{1}{2}\left(aW(\boldsymbol{\tau}^0) - aW(\tilde{\tilde{\boldsymbol{\tau}}})\right) > W_1(\tilde{\boldsymbol{\tau}}),$$

the government never sets $\tilde{\boldsymbol{\tau}}$ as it does not get any contributions for this policy. Similarly to above, the government prefers tariff $\boldsymbol{\tau}^0$ to $\tilde{\tilde{\boldsymbol{\tau}}}$. Therefore, by increasing $\widetilde{\widetilde{B_1}}$ industry 1 receives a payoff $W_1(\boldsymbol{\tau}_0) < \widetilde{\widetilde{B_1}}$, so this cannot be a best response.