Trade Liberalization and Productivity Growth

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April 25, 2007

Abstract: This paper presents a trade model with firm-level productivity differences and R&D-driven growth. Trade liberalization causes the least productive firms to exit but also slows the development of new products. The overall effect on productivity growth depends on the size of intertemporal knowledge spillovers in R&D. When these spillovers are relatively weak, then trade liberalization promotes productivity growth in the short run and makes consumers better off in the long run. However, when these spillovers are relatively strong, then trade liberalization retards productivity growth in the short run and makes consumers worse off in the long run.

Keywords: International Trade, Trade Liberalization, Productivity Growth,
Heterogeneous Firms, Endogenous Growth.

JEL: F12, F13, O31, O41

Acknowledgements: We thank Philippe Aghion, Josef Falkinger, Sara Formai, Rikard Forslid, Martin Flodén, Richard Kneller, Damian Migueles, Frédéric Robert-Nicoud, Ignat Stepanok, Fabrizio Zillibotti, Josef Zweimüller and seminar participants at the Stockholm School of Economics, Universite Catholique de Louvain, University of Nottingham and University of Zurich for helpful comments. Financial support from the Wallander Foundation is gratefully acknowledged.

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1 Introduction

Empirical research has established that there are large and persistant productivity differences among firms in narrowly defined industries (Bartelsman and Doms 2000). Furthermore, these productivity differences are important for understanding international trade. Even in so-called export sectors, many firms do not export their products and it is the most productive firms that tend to export. Trade liberalization induces the least productive firms to exit and induces more productive non-exporting firms to become exporters, market share reallocations that contribute in a significant way to productivity growth (i.e., Clerides, Lach and Tybout 1998, Bernard and Jensen 1999ab, Aw, Chung and Roberts 2000, Pavcnik 2002; see Tybout 2003 for a survey).

This evidence goes against both old trade theory (the Heckscher-Ohlin and Ricardian trade models) and the so-called new trade theory (Helpman and Krugman 1985). Both classes of trade models have representative firms and assume away any firm-level differences within sectors.

In response to the conflict between old theory and new evidence, a variety of trade models have been developed recently with firm-level productivity differences. Important contributions include Melitz (2003), Bernard, Eaton, Jensen and Kortum (2003), Helpman, Melitz and Yeaple (2004), Melitz and Ottaviano (2005), Yeaple (2005) and Baldwin (2005). These models can account for many of the new firm-level facts and represent significant advances in the theory of international trade. But one thing that is missing from all of these models is steady-state productivity growth. For example, in Melitz (2003), trade liberalization causes a permanent increase in productivity but the steady-state rate of productivity growth is zero.

In an important recent paper, Baldwin and Robert-Nicoud (2006) (henceforth referred to as BRN) develop a trade model with both firm-level productivity differences and steady-state productivity growth. They find that introducing an engine of growth into the Melitz (2003) model makes a big difference. Melitz established that trade liberalization promotes productivity growth in the short run and makes consumers better off in the long run. In contrast, assuming the same R&D technology that drives productivity growth in Grossman and Helpman (1991), BRN find that trade liberalization permanently retards productivity growth and makes consumers worse off in the long run. Central implications of Melitz (2003) are reversed.

One drawback of the BRN model is that the steady-state rate of productivity growth is an increasing function of population size, which implies that larger economies should grow faster. This "strong scale effect" property is present in all the early R&D-driven endogenous growth models, including Grossman and Helpman (1991), but is clearly in conflict with empirical evidence. As Jones (1995a) has pointed out, there have been no upward trends in the productivity growth rates of the United States, France, Germany or Japan since 1950 in spite of substantial increases in population size and R&D employment.[1] In response to the Jones critique, a variety of R&D-driven growth models have been developed that do not have the strong scale effect property, including Jones (1995b), Kortum (1997), Young (1998), Segerstrom (1998) and Howitt (1999).

In this paper, we revisit the question: what are the effects of trade liberalization when there are firm-level productivity differences? We present an improved version of the Melitz (2003) model with steady-state productivity growth but without the strong scale effect. The engine of growth is the introduction of new products, which is modelled in the same way as in Jones (1995b). Although our model is structurally quite similar to BRN (their Grossman-Helpman version), we find that getting rid of the strong scale effect significantly changes both the equilibrium and the welfare implications of trade liberalization. For a wide range of parameter values (not all), trade liberalization promotes productivity growth in the short run and makes consumers better off in the long run. Trade liberalization never permanently reduces the productivity growth rate (as BRN found) but for some parameter values, it does

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[1] For example, during the time period 1950-1987, the number of scientists and engineers engaged in R&D in the US grew from less than 200,000 to almost one million, a more than five-fold increase. The steady-state productivity growth rate is proportional to the R&D employment level in Grossman and Helpman (1991), so an increase in population size that leads to a five-fold increase in R&D employment should also lead to a five-fold increase in productivity growth.

make consumers worse off in the long run.

In the model, firms do R&D to develop new products and then learn how costly it is to produce these new products. Once firms have learned what their marginal costs of production are, they decide whether or not to incur the one-time fixed costs of entering the local and foreign markets. The fixed cost of entering the foreign market is assumed to be higher, consistent with the evidence in Roberts and Tybout (1997ab), Bernard and Jensen (2001) and Bernard and Wagner (2001) that potential exporters face significant foreign-market entry costs.

The model has a unique symmetric steady-state equilibrium where firms that develop new products with high marginal costs of production immediately exit. Firms that develop new products with intermediate marginal costs incur the fixed cost of entering the local market and only firms with sufficiently low marginal costs choose to also incur the fixed cost of entering the foreign market. Consistent with the empirical evidence, many firms do not export their products and it is the most productive (lowest marginal cost) firms that export in steady-state equilibrium.

We find that trade liberalization always causes the least productive firms to exit, which by itself contributes to productivity growth. In addition, trade liberalization induces the more productive non-exporting firms to incur the fixed cost of entering the foreign market, increases the expected fixed costs of developing profitable new products and causes a temporary slowdown in the development of new products. The overall effect of trade liberalization on productivity growth depends on whether intertemporal knowledge spillovers in R&D are relatively weak or relatively strong. When these spillovers are relatively weak, then trade liberalization promotes productivity growth in the short run and makes consumers better off in the long run. However, when these spillovers are relatively strong, then trade liberalization retards productivity growth in the short run and makes consumers worse off in the long run. BRN do not allow for any variation in the strength of intertemporal knowledge spillovers in R&D and implicitly assume that these spillovers are quite strong.

So are intertemporal knowledge spillovers in R&D relatively weak or relatively strong? We present empirical evidence that points to the first case (relatively weak spillovers) as being the more relevant one. In particular, we show that the first case occurs whenever the development of new products becomes more difficult over time.

The rest of the paper is organized as follows: in section 2, the dynamic general equilibrium trade model is presented and in section 3, it is solved for a unique symmetric steady-state equilibrium. Section 4 studies the implications of trade liberalization and section 5 concludes.

2 The Model

2.1 Overview of the model

In the model, there are two symmetric economies (or countries), a single primary factor labor that is inelastically supplied, a single consumption-good sector where there is Dixit-Stiglitz monopolistic competition and a single innovation sector where firms create knowledge through R&D.

Each firm's cost function is linear and involves one-time fixed costs and constant marginal production costs. To produce, a firm must first develop a new variety and this is done in the innovation sector. There is a one-time variety-development fixed cost which represents the sunk cost of developing a new variety. After having incurred this fixed cost, the firm receives a patent to exclusively produce the new variety and learns the unit labor requirement associated with its production. The unit labor requirement is drawn from a probability density function, so different firms have different marginal costs of production.

In addition to the above-mentioned costs, there are also market-entry costs. After having developed a new variety and learned its unit labor requirement, a firm decides whether or not to incur the one-time fixed costs of selling the variety in the local and foreign markets. We think of these market-entry costs as reflecting the costs of adapting the variety to market-specific standards, regulations and norms. A firm needs to draw a sufficiently low unit labor requirement to justify entering the local market and an even more favorable draw to justify entering the foreign market (see Figure 1). Roberts and Tybout (1997a) (for Colombia), Bernard and Jensen (2001) (for the U.S.) and Bernard and Wagner (2001) (for Germany) all find that potential exporters face significant foreign-market entry costs.

For firms that incur the one-time foreign-market entry cost and learn how to export, there are iceberg trade costs associated with shipping their products to the foreign market. The main focus of this paper is on exploring the steady-state equilibrium and welfare implications of trade liberalization, that is, a decrease in these iceberg trade costs and/or the costs of entering foreign markets.

Since the model consists of two trading economies (or countries) that are structurally symmetric, the choices made by agents in both economies can be understood by focusing on one of the two economies. This will be done throughout the rest of the paper.

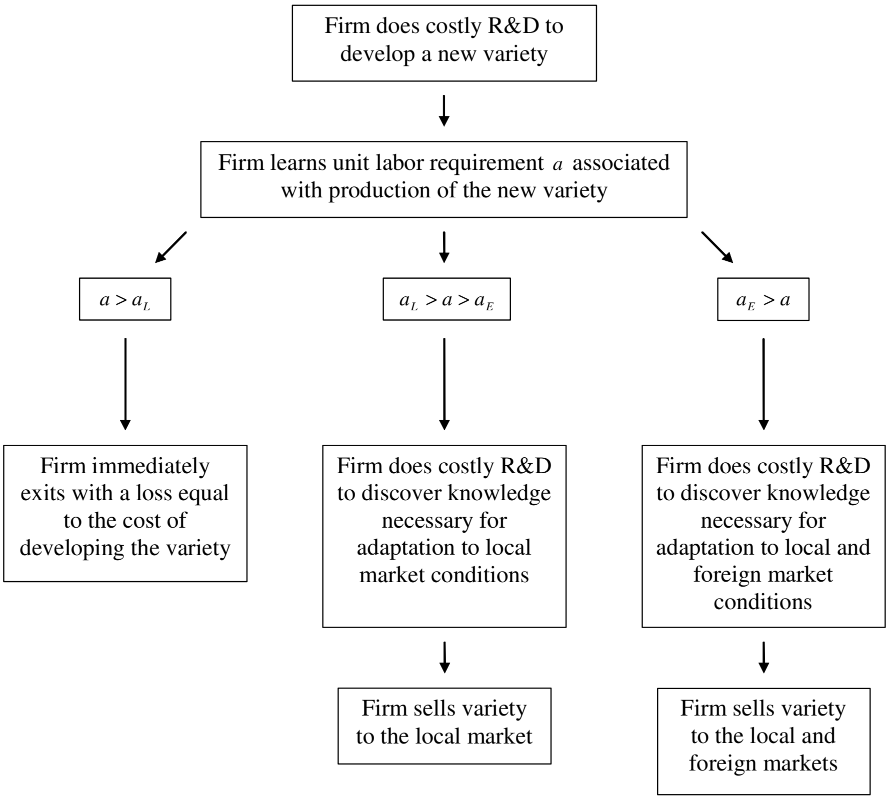

Figure 1. Description of the model.

2.2 Labor

In each economy, there is a fixed measure of households that provide labor services in exchange for wage payments. Each individual member of a household lives forever and is endowed with one unit of labor which is inelastically supplied. The size of each household, measured by the number of its members, grows exponentially at the exogenous population growth rate $n > 0$. Let $L_t = L_0 e^{nt}$ denote the supply of labor in each economy at time t.

The market for labor is perfectly competitive in each economy. Labor is employed either in the production of varieties or in the innovation sector doing R&D. Hence

$$L_t = L_{Pt} + L_{It}$$

where L_{Pt} is the economy-wide amount of labor used to produce varieties and L_{It} is the economy-wide amount of labor used in the innovation sector. Labor is perfectly mobile within an economy and is paid the common wage rate w per unit of labor supplied. The wage rate w is the same in both economies by symmetry and we set $w = 1$, treating labor as the numeraire.

2.3 Consumption

Households share identical preferences. Each household is modelled as a dynastic family that maximizes discounted lifetime utility

$$U = \int_0^\infty e^{-(\rho-n)t} \ln\left[u_t\right] dt \tag{1}$$

where $\rho > n$ is the subjective discount rate and u_t is the instantaneous utility of an individual household member at time t. The representative consumer has a C.E.S. utility function given by

$$u_t = \left[\int_0^{m_t^c} x_t(\omega)^\alpha \, d\omega \right]^{\frac{1}{\alpha}} \qquad 0 < \alpha < 1 \tag{2}$$

where $x_t(\omega)$ is the consumer's quantity consumed of a product ω at time t and m_t^c is the number of available varieties in an economy at time t (both domestically produced and imported varieties). The parameter α measures the degree of product differentiation. We assume that products are substitutes, which implies that $0 < \alpha < 1$ and yields an elasticity of substitution between any two products of $\sigma \equiv \frac{1}{1-\alpha} > 1$.

Solving the static optimization problem yields the familiar demand function

$$x_t(\omega) = \frac{p_t(\omega)^{-\sigma}}{\int_0^{m_t^c} p_t(\omega)^{1-\sigma} \, d\omega} E_t \tag{3}$$

where E_t is individual consumer expenditure and $p_t(\omega)$ is the price charged for product ω at time t. Furthermore, if we let

$$P_t = \left(\int_0^{m_t^c} p_t(\omega)^{1-\sigma} \, d\omega \right)^{\frac{1}{1-\sigma}}$$

denote the aggregate price index and let $c_t \equiv \frac{E_t}{P_t}$ be a measure of real consumption expenditure, then Dixit and Stiglitz (1977) have shown that $u_t = c_t$, that is, each consumer's instantaneous utility coincides with their real consumption expenditure.

Taking prices and expenditure as given, dynamic optimization yields the usual Euler equation

$$\frac{\dot{E}_t}{E_t} = r_t - \rho. \tag{4}$$

Individual consumer expenditure E_t grows over time if and only if the market interest rate r_t exceeds the subjective discount rate ρ.

2.4 Innovation

In the innovation sector, firms create knowledge by doing R&D. The unit labor requirement associated with creating knowledge is b_{It}, that is, it takes b_{It} units of labor at time t to create one unit of knowledge. Individual firms treat b_{It} as a parameter but it can change over time due to knowledge spillovers.

Following Jones (1995b), we assume that

$$b_{It} = \frac{1}{(m_{Lt} + \lambda m_{Ft})^{\phi}}$$

where $\phi < 1$ and $\lambda \in [0, 1]$ are given R&D parameters, m_{Lt} and m_{Ft} are the number of varieties produced in the local and foreign markets, ϕ measures the strength of intertemporal knowledge spillovers and λ measures the international dimension of spillovers. $\lambda = 0$ corresponds to no international spillovers and $\lambda = 1$ corresponds to perfect international spillovers. We allow for all the inbetween possibilities. Given symmetry, $m_{Lt} = m_{Ft} \equiv m_t$ where m_t is the number of varieties produced per economy and hence

$$b_{It} = \frac{1}{(1 + \lambda)^{\phi} m_t^{\phi}}. \tag{5}$$

The parameter ϕ is a key parameter in the model. We impose the restriction $\phi < 1$ to rule out explosive growth, as in Jones (1995b). Nevertheless, $\phi < 1$ allows for a wide range of values for the degree of intertemporal knowledge spillovers. In particular, these spillovers can be either positive or negative. For $\phi > 0$, researchers become more productive in creating new knowledge as the stock of knowledge measured by m_t increases over time. Researchers experience a "standing on the shoulders" effect. For $\phi < 0$, researchers become less productive in creating new knowledge as the stock of knowledge increases over time. Researchers experience a "fishing out" effect.[2]

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[2] Coe and Helpman (1995) argue that knowledge spillovers are related to trade flows; see Keller (2002) for a critique. We have studied how our model's properties change when knowledge spillovers depend on the volume of trade. We find that the results derived in this paper are qualitatively robust to this modification.

To develop a new variety, a firm needs to create F_I units of knowledge in the innovation sector. Thus, the cost of developing a new variety is $b_{It}F_I$ at time t. Knowledge creation is also involved in adapting a variety to market-specific standards, regulations and norms. To sell a new variety in the local market, a firm needs to create F_L units of knowledge at cost $b_{It}F_L$ and to sell a new variety in the foreign market, a firm needs to create F_E units of knowledge at cost $b_{It}F_E$.

Once a firm has developed a new variety, it learns the unit labor requirement a associated with its production. The unit labor requirement a is drawn from a probability density function $g(a)$ with support $[0, \overline{a}]$ and corresponding cumulative distribution function $G(a)$. Once drawn, the unit labor requirement of a firm associated with producing a particular variety does not change over time. We assume that the probability distribution of unit labor requirements is Pareto, that is,

$$G(a) = \int_0^a g(a)da = \left(\frac{a}{\overline{a}}\right)^k, \ a \in [0, \overline{a}]$$

where k and $\overline{a}$ are the shape and scale parameters of the distribution. Melitz (2003) worked with a general probability distribution of unit labor requirements but the model becomes considerably more tractable analytically if a Pareto distribution is assumed. The empirical literature on firm size distribution suggests that a Pareto distribution is a reasonable approximation (Del Gatto et al, 2006).[3]

Because of the heterogeneity in unit labor requirements, the model generates three types of firms: non-producing firms, local firms and exporting firms. Firms that get sufficiently unfavorable draws choose not to produce, firms that get intermediate draws choose to just produce for the local market, and firms that get the most favorable draws choose to produce

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In particular, the larger is the parameter that determines the size of the spillovers related to trade flows, the larger is the range of other parameter values for which trade liberalization promotes productivity growth in the short run.

[3] Usually the Pareto distribution is defined by $P(X > x) = (x/\overline{x})^{-k}$. Substituting $a = 1/x$ and $\overline{a} = 1/\overline{x}$ yields $G(a) = (a/\overline{a})^k$.

for both the local and the foreign markets.

For a firm that develops a new variety at time t, let a_{Lt} denote the unit labor requirement (or marginal cost) at which the firm is indifferent between incurring the fixed cost $b_{It}F_L$ of selling in the local market and immediately shutting down production. Similarly, let a_{Et} denote the unit labor requirement (or marginal cost) at which the firm is indifferent between selling in the local market only and incurring the additional fixed cost $b_{It}F_E$ to export its variety. We will solve the model for a steady-state equilibrium where the threshold values a_{Lt} and a_{Et} satisfy

$$0 < a_{Et} < a_{Lt} < \overline{a} \text{ for all } t,$$

that is, not all firms export ($a_{Et} < a_{Lt}$) and not all firms produce ($a_{Lt} < \overline{a}$).

2.5 Product markets

Given a draw of $a \leq a_{Lt}$ from the common density function $g(a)$, a firm's profits from local sales of variety ω are given by

$$\pi_{Lt}(\omega) = \max_{p_{Lt}(\omega)} \left(p_{Lt}(\omega) - a(\omega)\right) x_{Lt}(\omega)$$

where $x_{Lt}(\omega)$ is the demand for a locally sold variety ω and $p_{Lt}(\omega)$ is the corresponding price. $x_{Lt}(\omega)$ is given by (3) where E_t is redefined to measure the economy-wide consumer expenditure. Since there exists a continuum of firms, each firm chooses a profit-maximizing price taking aggregate expenditure and other firms' prices as given. This yields

$$p_L(\omega) = \frac{\sigma}{\sigma - 1} a(\omega)$$

which is the standard markup of price over marginal cost $a(\omega)$, given that $\sigma > 1$. By substituting for the price, profits of a firm selling locally can be written as

$$\pi_{Lt}(\omega) = (\sigma - 1)^{\sigma-1} \sigma^{-\sigma} \left(\frac{a(\omega)}{P_t} \right)^{1-\sigma} E_t. \tag{6}$$

Similarly, given a draw $a \leq a_{Et}$, additional profits from exports are given by

$$\pi_{Et}(\omega) = \max_{p_{Et}(\omega)} \left(p_{Et}(\omega) - \tau a(\omega) \right) x_{Et}(\omega)$$

where $x_{Et}(\omega)$ is the foreign demand for the exported variety ω and $p_{Et}(\omega)$ is the price of the exported variety ω. $\tau > 1$ is an iceberg trade cost such that τ units must be shipped for one unit to reach its destination. The corresponding profit-maximizing price from foreign sales is given by

$$p_{Et}(\omega) = \frac{\sigma}{\sigma - 1} \tau a(\omega).$$

Hence, an exporting firm sets a higher price on varieties sold to foreign consumers to compensate for the trade cost. Substituting for the price, the additional profits earned by an exporting firm are

$$\pi_{Et} = \theta (\sigma - 1)^{\sigma-1} \sigma^{-\sigma} \left(\frac{a(\omega)}{P_t} \right)^{1-\sigma} E_t, \tag{7}$$

where $\theta \equiv \tau^{1-\sigma}$ is a measure of the free-ness of trade ($\theta = 0$ describes the case of autarky, whereas $\theta = 1$ implies free trade).

2.6 Local and Foreign Market Entry

Having solved for the profits that firms earn from selling locally and exporting, we can now determine when firms choose to enter the local and foreign markets.

For the owners of a firm with marginal cost of production equal to either of the two threshold values ($a = a_{it}$, $i = L, E$), profits $\pi_{it}(a_{it})dt$ are earned during the time interval dt and the capital gain $\dot{V}_{it}(a_{it})dt$ is also realized, where $V_{it}(a_{it})$ is the discounted profit associated with the draw $a = a_{it}$. As there is no risk for the owners of a firm once it's a value is known, the total return on equity claims must equal the risk-free market interest rate r_t, that is

$$\pi_{it}(a_{it})dt + \dot{V}_{it}(a_{it})dt = r_t V_{it}(a_{it})dt \qquad i = L, E.$$

Solving for V_{it} yields

$$V_{it}(a_{it}) = \frac{\pi_{it}(a_{it})}{r_t - \frac{\dot{V}_{it}(a_{it})}{V_{it}(a_{it})}} \qquad i = L, E.$$

Note that, unlike in Melitz (2003), firms face no exogenous death rate. Melitz assumes a common exogenous death rate for all firms to enable transitions between steady-states (in response to trade liberalization). We can dispense with this assumption since variety innovation plays the same role of enabling transitions between steady-states.[4] However, all the results in the paper continue to hold when there is a common exogenous death rate for firms.

Given that the firms with threshold values a_{Lt} and a_{Et} are indifferent between entering and not entering the local and foreign markets respectively, the costs of entering must be exactly balanced by the benefits of entering:

$$V_{it}(a_{it}) = b_{It} F_i \qquad i = L, E.$$

Substituting into these two equations using the profit flows (6) and (7) yields the local market entry condition

$$\frac{(\sigma - 1)^{\sigma-1} \sigma^{-\sigma} \left(\frac{a_{Lt}}{P_t}\right)^{1-\sigma} E_t}{r_t - \frac{\dot{b}_{It}}{b_{It}}} = b_{It} F_L \tag{8}$$

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[4] As the number of varieties produced increases over time, the significance of firms that made entry decisions under the old regime (before trade liberalization) decreases and eventually becomes negligable.

and the foreign market entry condition

$$\frac{(\sigma-1)^{\sigma-1}\sigma^{-\sigma}\left(\frac{a_{Et}}{P_t}\right)^{1-\sigma}E_t}{r_t-\frac{\dot{b}_{It}}{b_{It}}}=\frac{b_{It}F_E}{\theta}. \tag{9}$$

For the model to be consistent with the observation that not all firms export ($a_{Lt} > a_{Et}$), we assume that the costs associated with foreign entry are higher than for local entry, that is, $F_E > F_L$. With this parameter restriction, (8) and (9) together imply that

$$\frac{a_{Lt}}{a_{Et}}=\left(\frac{F_E}{F_L\theta}\right)^{\frac{1}{\sigma-1}}>1 \tag{10}$$

for all $\theta \in (0,1)$. Firms that draw $a \leq a_{Lt}$ choose to enter the local market and firms that draw $a \leq a_{Et}$ choose to enter the foreign market as well. Firms that are unlucky and draw $a > a_{Lt}$ choose not to enter either the local or foreign markets.

2.7 Innovation Incentives

Having solved for when firms choose to sell locally and export, we can now work backwards and determine the incentives to develop new varieties.

We assume that there is free entry by firms into variety innovation. Since any firm can develop a new variety, the ex-ante expected benefit of developing a new variety must equal the cost of variety innovation. This can be stated as

$$\int_0^{a_{Lt}}\left\{\frac{\pi_{Lt}(a)}{r_t-\frac{\dot{b}_{It}}{b_{It}}}-b_{It}F_L\right\}dG(a)+\int_0^{a_{Et}}\left\{\frac{\pi_{Et}(a)}{r_t-\frac{\dot{b}_{It}}{b_{It}}}-b_{It}F_E\right\}dG(a)=b_{It}F_I$$

where $G(a)$ is the Pareto cumulative distribution function from which a potential market entrant draws a unit labor requirement. Substituting for profits using (6) and (7) and integrating, free entry ensures that ex-ante expected discounted profits must equal ex-ante expected fixed costs of developing a profitable variety:

$$\frac{(\sigma-1)^{\sigma-1}\sigma^{-\sigma}E_t\Delta_t}{P_t^{1-\sigma}\left(r_t-\frac{\dot{b}_{It}}{b_{It}}\right)}=b_{It}\overline{F}_t \tag{11}$$

where

$$\Delta_t\equiv\int_0^{a_{Lt}}a^{1-\sigma}\frac{g(a)}{G(a_{Lt})}da+\theta\int_0^{a_{Et}}a^{1-\sigma}\frac{g(a)}{G(a_{Lt})}da \tag{12}$$

is a weighted average of firms' productivities and

$$\overline{F}_t\equiv F_I\frac{1}{G(a_{Lt})}+F_L+F_E\frac{G(a_{Et})}{G(a_{Lt})}.$$

Using the properties of the Pareto distribution, $\overline{F}_t$ can be written as

$$\overline{F}_t=F_I\left(\frac{\overline{a}}{a_{Lt}}\right)^k+F_L+F_E\left(\frac{a_{Et}}{a_{Lt}}\right)^k. \tag{13}$$

The first term on the right-hand-side of (13) represents the expected cost of innovation, where $\frac{1}{G(a_{Lt})}=\left(\frac{\overline{a}}{a_{Lt}}\right)^k$ can be seen as the number of attempts needed before a profitable variety is discovered. F_L is the fixed cost of local market adaptation paid by all producing firms. The third term is the expected fixed cost associated with adapting a variety to the foreign market. $\left(\frac{a_{Et}}{a_{Lt}}\right)^k=\frac{G(a_{Et})}{G(a_{Lt})}$ represents the likelihood of having developed a variety profitable enough to export, given that local market entry has taken place. Hence, $\overline{F}_t$ is the ex ante expected fixed cost of developing a profitable variety at time t, measured in units of knowledge created.[5]

The flow of new varieties is determined by the labor devoted to R&D divided by the

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[5] For a firm that has developed a new variety, it is straightforward to show that delaying market entry is never profit-maximizing. In cases where the costs of market entry fall by waiting, the benefits of market entry also fall by waiting. If it is profitable for a firm to enter a market, then it wants to do so as soon as possible. Delaying market entry always "leaves money on the table".

labor units required for successful innovation:

$$\dot{m}_t = \frac{L_{It}}{b_{It}\overline{F}_t}, \tag{14}$$

where $L_{It} = \sum_i l_{Iit}$ is the sum of all R&D done by firms in the economy.

This completes the description of the model.

3 Solving the model

In this section, we solve the model for a steady-state equilibrium. A steady-state (or balanced growth) equilibrium is defined as an equilibrium where all endogenous variables grow at constant (not necessarily identical) rates over time. Given the model's symmetric structure, we restrict attention to symmetric equilibrium outcomes.

Equation (4) implies that the market interest rate r_t must be constant over time in any steady-state equilibrium. It then follows from studying (11) that, as the growth rates of the endogenous variables P_t, E_t, Δ_t, and b_{It} are all constant over time, $\overline{F}_t$ must grow at a constant rate. But as F_L is a constant, (13) implies that $\overline{F}_t$ cannot grow at a constant rate unless $a_{Lt} = a_L$ and $a_{Et} = a_E$ for all t. Hence $\overline{F}$, a_L and a_E are all constants in any steady-state equilibrium.

Let $g \equiv \frac{\dot{m}_t}{m_t}$ denote the steady-state rate of innovation. Dividing both sides of (14) by m_t, and substituting for b_{It} using (5) yields

$$g \equiv \frac{\dot{m}_t}{m_t} = \frac{L_{It}\,(1+\lambda)^{\phi}}{\overline{F} m_t^{1-\phi}}.$$

Because labor supply L_t grows at the constant exogenous rate n and consists of workers employed in either production or R&D, R&D labor L_{It} must grow at the rate n as well. It then follows that g can only be constant over time if

$$g = \frac{n}{1-\phi}. \tag{15}$$

Equation (15) establishes that the steady-state rate of innovation g is proportional to the population growth rate n. As in Jones (1995b), the parameter restriction $\phi < 1$ is needed to guarantee that the steady-state rate of innovation is positive and finite (given that there is positive population growth).

Equation (15) has two important implications. First, it implies that public policy changes like trade liberalization have no effect on the steady-state rate of productivity growth (note that τ or F_E do not appear in the equation). We view this as a virtue of the model because both total factor productivity and per capita GDP growth rates have been remarkably stable over time in spite of many public policy changes that one might think would be growth-promoting. For example, plotting data on per capita GDP (in logs) for the US from 1880 to 1987, Jones (1995a) shows that a simple linear trend fits the data extremely well. Second, equation (15) implies that the level of per capita income in the long run is an increasing function of the size of the economy. Jones (2005b) has a lengthy discussion of this "weak scale effect" property and cites Alcala and Ciccone (2004) as providing the best empirical support. Controlling for both trade and institutional quality, Alcala and Ciccone (2004) find that a 10 percent increase in the size of the workforce in the long run is associated with 2.5 percent higher GDP per worker.[6]

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[6] One piece of evidence that is often misinterpreted as going against the weak scale effects prediction is the negative coefficient on population growth in typical cross-country growth regressions, such as in Mankiw, Romer and Weil (1992). Other things being equal, countries with higher population growth rates tend to have lower per capita incomes. This evidence is addressed in Jones (2005b) and his response is worth quoting: "Recall that the standard interpretation of these regressions is that they are estimating transition dynamics. The negative coefficient on population growth is interpreted as capturing the dilution of the investment rate associated with the Solow model. Consider two countries that are identical but for different population growth rates. The country with the faster population growth rate must equip a larger number of new workers with the existing capital-labor ratio, effectively diluting the investment rate. The result is that such an economy has a lower capital-output ratio in steady-state, reducing output per worker along the balanced growth path. But this same force is also at work in any growth model, including idea-based models, as was apparent above in Result (1b). The implication is that this cross-country evidence is not inconsistent with models in which weak scale effects play a role."

The R&D technology used in this paper differs from the ones used in BRN (2006), who study the implications of five alternative R&D technologies. To facilitate comparison with their paper, we focus on their first model ("the Grossman-Helpman model") although the points that we make apply with appropriate modification to their other models as well. BRN focus on the special case where the intertemporal knowledge spillover parameter ϕ equals one. The assumption $\phi = 1$ implies that the unit labor requirement associated with creating knowledge is

$$\widetilde{b}_{It} = \frac{1}{(1+\lambda)\,\widetilde{m}_t}$$

where $\widetilde{\ }$ is used to distinguish variables in BRN (2006) from corresponding variables in this paper. The number of innovated varieties then increases over time according to

$$\dot{\widetilde{m}}_t = \frac{\widetilde{L}_{It}\,(1+\lambda)\,\widetilde{m}_t}{\overline{F}}$$

and the corresponding rate of innovation is given by

$$\widetilde{g} \equiv \frac{\dot{\widetilde{m}}_t}{\widetilde{m}_t} = \frac{(1+\lambda)}{\overline{F}}\widetilde{L}_{It}.$$

Hence, in contrast to this paper, the choice of R&D technology made by BRN implies that the rate of innovation g is proportional to the R&D employment level L_{It}. BRN then proceed to show that the steady-state economic growth rate is also proportional to the level of R&D employment.

This strong scale effect property is clearly at odds with the available empirical evidence. Between 1950 and 1993, the number of scientists and engineers engaged in research in the G-5 countries (France, Germany, Japan, the United Kingdom and the United States) rose by more than a factor of eight without generating any upward trend in productivity growth rates (Jones, 2005b). To rule out the strong scale effect, we assume a modified R&D technology where $\phi < 1$.[7]

Returning to the model, we solve next for the steady-state profit rates. We first note that $\Delta_t = \Delta$ for all t according to (12) as the unit labor requirement thresholds a_L and a_E are constant in steady-state. To solve for the aggregate price index, we first note that

$$P_t^{1-\sigma} = \int_0^{m_t^c} p(\omega)^{1-\sigma} d\omega = \int_0^{a_L} p_L(a)^{1-\sigma} m_{Lt} \frac{g(a)}{G(a_L)} da + \int_0^{a_E} p_E(a)^{1-\sigma} m_{Ft} \frac{g(a)}{G(a_L)} da$$

where m_{Lt} is the number of locally produced varieties, m_{Ft} is the number of foreign produced varieties and $\frac{g(a)}{G(a_L)}$ is the steady-state density function conditional on entry. Substituting for prices, the aggregate price index is given by

$$P_t = \left(\frac{\sigma}{\sigma - 1}\right) (m_t \Delta)^{\frac{1}{1-\sigma}} . \tag{16}$$

This implies that the profits earned from selling locally (6) and exporting (7) can be rewritten as

$$\pi_{Lt} = s_t(a) \frac{E_t}{\sigma} \tag{17}$$

$$\pi_{Et} = \theta s_t(a) \frac{E_t}{\sigma} \tag{18}$$

where $s_t(a) \equiv \frac{a^{1-\sigma}}{m_t \Delta}$ is the Dixit-Stiglitz market share at time t.

To solve for a steady-state equilibrium, we need to define the concept of relative R&D difficulty:

$$z_t \equiv \frac{m_t^{-\phi}}{\frac{L_t}{m_t}} = \frac{m_t^{1-\phi}}{L_t}. \tag{19}$$

In (19), $m_t^{-\phi}$ is a measure of absolute R&D difficulty and $\frac{L_t}{m_t}$ is a measure of the size of the market for each variety. Thus relative R&D difficulty is R&D difficulty relative to the size

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[7] The strong scale effect is present in all first generation R&D-driven endogenous growth models, including Romer (1991), Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992).

of the market. By log-differentiating (19) and using (15), we have that

$$\frac{\dot{z}_t}{z_t} = (1-\phi)\,g - n = 0.$$

Hence, relative R&D difficulty z is constant in steady-state equilibrium.[8]

Having defined z, we can state concisely the steady-state market entry conditions. From (4), we note that since the growth rate of consumer expenditure must be constant in any steady-state equilibrium, the market interest rate must be constant as well, so $r_t = r$ for all t. Taking logs and differentiating (5) with respect to time yields $\frac{\dot{b}_{It}}{b_{It}} = -\phi g$. Also using (5) to substitute for b_{It}, the local market entry condition (8) can now be written as

$$\frac{\frac{a_L^{1-\sigma}}{\Delta}\frac{E_t}{\sigma}}{r+\phi g} = \frac{F_L z L_t}{(1+\lambda)^{\phi}}. \tag{20}$$

The corresponding foreign market entry condition given by (9) is

$$\frac{\frac{a_E^{1-\sigma}}{\Delta}\frac{E_t}{\sigma}}{r+\phi g} = \frac{F_E z L_t}{\theta\,(1+\lambda)^{\phi}} \tag{21}$$

where the left-hand-sides of (20) and (21) are associated with the discounted benefits of a firm having drawn $a = a_L$ or $a = a_E$, while the corresponding right-hand-sides are associated with the costs of local or foreign market entry.

Using the definition of z and substituting for the aggregate price index P_t, the unit labor requirement for knowledge creation b_{It} and the capital gain term $\frac{\dot{b}_{It}}{b_{It}}$, the free entry into variety innovation condition (11) can be written concisely as

$$\frac{\frac{E_t}{\sigma}}{r+\phi g} = \frac{\overline{F} z L_t}{(1+\lambda)^{\phi}}. \tag{22}$$

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[8] The concept of relative R&D difficulty was introduced in Segerstrom (1998). It is defined differently in this paper because the number of varieties increases over time, which decreases the market for each individual variety.

This condition governs the steady-state incentives for developing new varieties since the left-hand-side is associated with the discounted benefits of developing and producing a profitable variety whereas the right-hand-side is associated with the expected costs of development and market entry.

Next, we turn to labor markets, which are assumed to be perfectly competitive. The total workforce used in industry production is given by the sum of the labor producing for the local market and the labor used to produce goods sold in the foreign market. To produce a variety that will be sold locally, a firm ω needs $a(\omega)x_{Lt}(\omega)$ units of labor. Substituting for demand and prices using (3) and (16), this corresponds to

$$a(\omega)x_{Lt}(\omega) = \left(\frac{\sigma-1}{\sigma}\right)\frac{a(\omega)^{1-\sigma}}{m_t\Delta_t}E_t.$$

To produce the goods that are exported, a firm ω needs $\tau a(\omega)x_{Et}(\omega)$ units of labor, which can be similarly shown to equal

$$\tau a(\omega)x_{Et}(\omega) = \theta\left(\frac{\sigma-1}{\sigma}\right)\frac{a(\omega)^{1-\sigma}}{m_t\Delta_t}E_t.$$

Summing over firms, the production labor of an economy in steady-state amounts to

$$L_{Pt} = \int_0^{a_L} a(\omega)x_{Lt}(\omega)m_t\frac{g(a)}{G(a_L)}da + \int_0^{a_E} \tau a(\omega)x_{Et}(\omega)m_t\frac{g(a)}{G(a_L)}da$$

where m_t represents the total number of varieties produced per country. Substituting for $a(\omega)x_{Lt}(\omega)$, $\tau a(\omega)x_{Et}(\omega)$ and using (12), this simplifies to $L_{Pt} = \left(\frac{\sigma-1}{\sigma}\right)E_t$. The full employment condition $L_t = L_{Pt} + L_{It}$ then implies that

$$E_t = L_t + \frac{E_t}{\sigma} - L_{It}. \tag{23}$$

Equation (23) has a simple economic interpretation. Aggregate expenditure E_t equals aggregate income, which is given by the sum of aggregate labor income L_t and aggregate profit income $\frac{E_t}{\sigma}$, deducting the wages paid in the innovation sector L_{It}. To see that aggregate profit income is $\frac{E_t}{\sigma}$, it suffices to verify that

$$\int_0^{a_L} \pi_L(a) m_t \frac{g(a)}{G(a_L)} da + \int_0^{a_E} \pi_E(a) m_t \frac{g(a)}{G(a_L)} da = \frac{E_t}{\sigma}$$

using (12), (17) and (18).

To solve for the steady-state interest rate r, we use (23) and note that with total labor L_t growing at the rate n, R&D labor L_{It} and aggregate expenditure E_t must grow at the rate n as well. Thus individual consumer expenditure must be constant over time and equation (4) implies that the steady-state interest rate is $r = \rho$.

The weighted average of firms' productivities Δ_t is constant in steady-state equilibrium and can be written much more simply. Taking into account that the a's are drawn from the Pareto distribution, (12) implies that

$$\Delta = \frac{a_L^{1-\sigma}}{1 - \frac{\sigma-1}{k}} \left(1 + \theta \left(\frac{a_E}{a_L} \right)^{1+k-\sigma} \right).$$

We assume that $\beta \equiv \frac{k}{\sigma-1} > 1$ to guarantee that Δ is finite. Then letting $T \equiv \frac{F_E}{F_L}$ and $\Omega \equiv \theta^{\beta} T^{1-\beta}$ and substituting using (10), we obtain that

$$\Delta = \frac{\beta}{\beta - 1} a_L^{1-\sigma} (1 + \Omega). \tag{24}$$

The variable Ω measures the degree of openness and depends on both the fixed and the variable trade costs (F_E and τ). Given that the inequality in (10) holds and not all firms export, Ω is bounded between zero and one. $\Omega = 0$ corresponds to infinite τ and/or infinite $\frac{F_E}{F_L}$ whereas $\Omega = 1$ corresponds to zero iceberg trade costs ($\tau = 1$) and $F_E = F_L$.

In steady-state equilibrium, the expected fixed cost of developing a profitable variety $\overline{F}$ is constant over time and depends on the degree of openness Ω. Substituting for Δ using

(24) in the the local market entry condition (20) and combining this with the innovation entry condition (22), $\overline{F}$ can be explicitly expressed as

$$\overline{F} = F_L \left(\frac{\beta}{\beta - 1} \right) (1 + \Omega) . \tag{25}$$

In steady-state equilibrium, aggregate expenditure E_t grows over time and we can solve for the exact path. First, (5), (14) and (19) together imply that

$$L_{It} = \frac{\overline{F} z L_t}{(1 + \lambda)^{\phi}} g. \tag{26}$$

Second, (22) can be solved for $\frac{E_t}{\sigma}$. Substituting these results into (23) and simplifying using (15), steady-state aggregate expenditure is found to be

$$E_t = L_t \left(1 + \frac{(\rho - n) \overline{F} z}{(1 + \lambda)^{\phi}} \right) . \tag{27}$$

Free entry ensures that the value of the average producing firm (expected discounted profits) must equal the expected fixed costs of developing a profitable variety

$$b_{It} \overline{F} = \frac{\overline{F}}{(1 + \lambda)^{\phi} m_t^{\phi}} .$$

Hence, the value of all the producing firms in the economy is $\frac{\overline{F} m_t^{1-\phi}}{(1+\lambda)^{\phi}} = \frac{\overline{F} z L_t}{(1+\lambda)^{\phi}}$ and equation (27) states that aggregate expenditure is equal to the sum of labor income and the returns from the ownership of firms.

All that remains in solving the model is to find steady-state values for a_L, a_E and z. First, we combine (10), (13) and (25) to obtain the steady-state cut-off unit labor requirement for selling locally:

$$a_L = \overline{a} \left(\frac{F_I (\beta - 1)}{F_L (1 + \Omega)} \right)^{\frac{1}{k}} . \tag{28}$$

To guarantee that $a_L < \overline{a}$, we assume that the fixed cost of entering the local market is sufficiently large, in particular, $F_L > F_I(\beta - 1)$. Next a_E is determined by using (28) to substitute for a_L in (10). The steady-state cut-off unit labor requirement for exporting is

$$a_E = \overline{a}\left(\frac{F_I(\beta-1)\Omega}{F_E(1+\Omega)}\right)^{\frac{1}{k}}. \tag{29}$$

Finally, using (25) and (27) to substitute for $\overline{F}$ and E_t in (22), we obtain steady-state relative R&D difficulty

$$z = \frac{(\beta-1)(1+\lambda)^{\phi}}{\beta F_L(1+\Omega)\left[(\sigma-1)\rho+\sigma\phi g+n\right]}. \tag{30}$$

Using (15), it is easy to verify that $z > 0$ for all $\phi < 1$ since $(\sigma-1)\rho+\sigma\phi g+n$ is globally increasing in ϕ and $\lim_{\phi\to-\infty}(\sigma-1)\rho+\sigma\phi g+n = (\sigma-1)(\rho-n) > 0$. We conclude that the model has a unique symmetric steady-state equilibrium.

4 Steady-state properties of the model

In this section the steady-state equilibrium implications of trade liberalization are studied. We suppose that the two trading economies are initially in steady-state equilibrium and that at some point in time trade liberalization takes place. By trade liberalization, we mean that Ω increases, which can be due to either a decrease in trade costs ($\tau \downarrow$) or a decrease in the knowledge needed to enter foreign markets ($F_E \downarrow$).

First, we focus on the implications of trade liberalization for market entry. From (28), an increase in Ω implies that a_L decreases. A decrease in the cut-off value for local market entry a_L means that fewer firms are willing to incur the fixed cost of local market entry and sell their product varieties in the local market. From (29), an increase in Ω also causes a_E to increase. An increase in the cut-off value for foreign market entry a_E means that more firms are willing to incur the fixed cost of foreign market entry and sell their product varieties in the foreign market. Furthermore, it is the least productive (highest marginal cost) local producers that become non-producers as a result of trade liberalization and it is the most productive (lowest marginal cost) non-exporters that become exporters as a result of trade liberalization. We have established

Theorem 1 *Trade liberalization* ($\Omega \uparrow$) *causes the least productive firms to exit* ($a_L \downarrow$) *and induces more firms to become exporters* ($a_E \uparrow$)

Theorem 1 is the same result as in Melitz (2003) but now it is derived from a model with steady-state productivity growth. In Melitz (2003), the steady-state equilibrium rate of productivity growth is zero.

In support of Theorem 1, several studies have established that exposure to trade opens up new growth opportunities for relatively productive firms. This pattern has been found in the U.S. by Bernard and Jensen (1999a), in Taiwan by Aw, Chung and Roberts (2000) and in Colombia, Mexico and Morocco by Clerides, Lach and Tybout (1998). In addition, the study by Aw, Chung and Roberts (2000) finds that the least productive firms exit as they are exposed to trade. By considering the direct effect from market share allocation on sectoral productivity growth, Pavcnik (2002) finds that the reallocations following trade liberalization in Chile had a significant impact on productivity growth in tradable sectors. In another related study, Bernard and Jensen (1999b) find that market share reallocation towards more productive exporting firms can explain as much as one fifth of U.S. manufacturing productivity growth.

Consider next the steady-state effects of trade liberalization for variety innovation and R&D behavior. From (15), an increase in Ω has no effect on $g \equiv \frac{\dot{m}_t}{m_t}$. In the long run, the growth rate of the number of varieties produced does not change as a result of trade liberalization. Turning next to the share of labor employed in R&D, substituting into (26) using (25) and (30) yields

$$\frac{L_{It}}{L_t} = \frac{g}{(\sigma - 1)\rho + \sigma\phi g + n}.$$

An increase in Ω has no effects on $\frac{L_{It}}{L_t}$ either. In the long run, trade liberalization has no effect on the share of labor employed in R&D. However, trade liberalization does affect z.

Equation (30) implies that an increase in Ω leads to a lower steady-state z. And from (19), the permanent reduction in z can only occur if m_t temporarily grows at lower rates than the steady-state growth rate $g \equiv \frac{\dot{m}_t}{m_t} = \frac{n}{1-\phi}$. Thus trade liberalization leads to a temporary slowdown in variety innovation. We have established

Theorem 2 *Trade liberalization* ($\Omega \uparrow$) *has no long-run effect on the growth rate of product variety* $\left(g \equiv \frac{\dot{m}_t}{m_t}\right)$ *or the share of labor devoted to R&D* $\left(\frac{L_{It}}{L_t}\right)$. *However, trade liberalization* ($\Omega \uparrow$) *does cause a temporary slowdown in variety growth* ($z \downarrow$) *and a permanent decrease in the number of varieties produced* ($m_t \downarrow$ *for all* t).

The intuition behind Theorem 2 is as follows. Trade liberalization (through a decrease in variable and/or fixed trade costs) makes it more profitable for firms to become exporters. When more firms become exporters, firms on average pay a higher total market entry cost $\overline{F}$ since more firms incur the cost of entering the foreign market ($\overline{F} \uparrow$ when $\Omega \uparrow$). As a result of the increase in the expected cost of innovation (including the market entry costs), the incentives to innovate are reduced and the rate of innovation drops below its steady-state rate g. One aspect of trade liberalization is that it involves a diversion of resources away from a growth-promoting activity: the development of new products.

By itself, Theorem 1 suggests that trade liberalization should promote productivity growth, at least in the short run. Trade liberalization causes the least productive firms to exit ($a_L \downarrow$) and also induces more productive firms to become exporters ($a_E \uparrow$). But Theorem 2 has implications for productivity growth that go in the opposite direction. Trade liberalization leads more firms to become exporters and this diverts resources away from product innovation, an activity that contributes to productivity growth. Thus, the overall effect of trade liberalization on productivity growth is not obvious.

To determine the overall effect of trade liberalization on productivity growth, we need to measure productivity and solve for how it evolves over time. Since $\frac{E_t}{P_t}$ is real output at time t and labor is the only factor of production, we use real output per worker $\frac{E_t}{P_t L_t}$ as our measure of productivity. Real output per worker coincides with per capita real consumption in the model and from section 2.3, this measure of productivity also coincides with consumer instantaneous utility ($u_t = c_t = \frac{E_t}{P_t L_t}$ where E_t is aggregate expenditure). Thus, the measure of productivity c_t is also a measure of consumer welfare.

Substituting for P_t using (16) and for $\frac{E_t}{L_t}$ using (27), productivity at time t can be written as

$$c_t = \left(1 + \frac{(\rho - n)\overline{F}z}{(1+\lambda)^{\phi}}\right)\left(\frac{\sigma - 1}{\sigma}\right)(m_t \Delta)^{\frac{1}{\sigma - 1}}. \tag{31}$$

In the model's steady-state equilibrium, $\overline{F}$, z and Δ are all constants over time. Thus productivity c_t grows over time only because the number of varieties m_t grows over time. Using (15), it immediately follows from (31) that

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma - 1}\frac{\dot{m}_t}{m_t} = \frac{n}{(\sigma - 1)(1 - \phi)}. \tag{32}$$

The steady-state growth rate of productivity $\frac{\dot{c}_t}{c_t}$ is proportional to the steady-state rate of innovation $\frac{\dot{m}_t}{m_t}$. Just as trade liberalization has no effect on the steady-state rate of innovation (Theorem 2), it is clear from (32) that trade liberalization has no effect on the steady-state rate of productivity growth.

Although trade liberalization does not have any long-run growth effects, it does have level effects on productivity and consumer welfare. To determine the direction of these level effects, we compare two steady-state equilibrium paths: one associated with an economy experiencing less restricted trade (Ω) and the other associated with an economy experiencing more restricted trade ($\Omega^* < \Omega$). To facilitate this comparison of steady-state paths, all variables associated with the more restricted trade path have the asterisk * attached, so in particular, c_t represents per capita real consumption on the steady-state path with less restricted trade (Ω) and c_t^* represents per capita real consumption on the steady-state path with more restricted trade ($\Omega^* < \Omega$). We solve for the ratio $\frac{c_t}{c_t^*}$. This ratio is constant over time since trade liberalization does not affect steady-state productivity growth. If $\frac{c_t}{c_t^*} > 1$, then we can conclude that trade liberalization ($\Omega^* \rightarrow \Omega$) promotes productivity growth in the short run and makes consumers better off in the long run. If instead $\frac{c_t}{c_t^*} < 1$, then we can conclude that trade liberalization retards productivity growth in the short run and makes consumers worse off in the long run.

To solve for $\frac{c_t}{c_t^*}$, first note that $\overline{F}z = \overline{F}^* z^*$ follows from (25) and (30). Equation (31) then implies that

$$\frac{c_t}{c_t^*} = \left(\frac{m_t \Delta}{m_t^* \Delta^*}\right)^{\frac{1}{\sigma-1}}.$$

Now (24) and (28) together imply that

$$\frac{\Delta}{\Delta^*} = \left(\frac{a_L}{a_L^*}\right)^{1-\sigma} \frac{1+\Omega}{1+\Omega^*} = \left(\frac{1+\Omega}{1+\Omega^*}\right)^{1+\frac{\sigma-1}{k}}$$

and (19) and (30) together imply that

$$\frac{m_t}{m_t^*} = \left(\frac{z}{z^*}\right)^{\frac{1}{1-\phi}} = \left(\frac{1+\Omega^*}{1+\Omega}\right)^{\frac{1}{1-\phi}}$$

Putting these results together yields

$$\left(\frac{c_t}{c_t^*}\right)^{\sigma-1} = \left(\frac{1+\Omega}{1+\Omega^*}\right)^{1+\frac{\sigma-1}{k}-\frac{1}{1-\phi}}$$

and $\frac{c_t}{c_t^*} > 1$ if and only if $1 + \frac{\sigma-1}{k} - \frac{1}{1-\phi} > 0$. This parameter condition simplifies to $\phi < \overline{\phi} \equiv \frac{1}{1+\beta}$ and thus we have established

Theorem 3 *If $\phi < \overline{\phi} \equiv \frac{1}{1+\beta}$, then trade liberalization promotes productivity growth in the short run and makes consumers better off in the long run $\left(\Omega > \Omega^* \Rightarrow \frac{c_t}{c_t^*} > 1\right)$. However, if $\phi > \overline{\phi} \equiv \frac{1}{1+\beta}$, then trade liberalization retards productivity growth in the short-run and makes consumers worse off in the long run $\left(\Omega > \Omega^* \Rightarrow \frac{c_t}{c_t^*} < 1\right)$.*

Theorem 3 is the main result in the paper and it is surprising. BRN found that trade liberalization permanently retards productivity growth and makes consumers worse off in the long run. Although our model is structurally quite similar to BRN (their Grossman-

Helpman version), we find that getting rid of the strong scale effect significantly changes both the equilibrium and the welfare implications of trade liberalization. For a wide range of parameter values ($\phi < 1/(1+\beta)$), trade liberalization promotes productivity growth in the short run and makes consumers better off in the long run. Only when intertemporal knowledge spillovers in R&D are sufficiently strong ($\phi > 1/(1+\beta)$) is this conclusion reversed.

Different possible effects of trade liberalization are illustrated in Figures 2, 3 and 4.

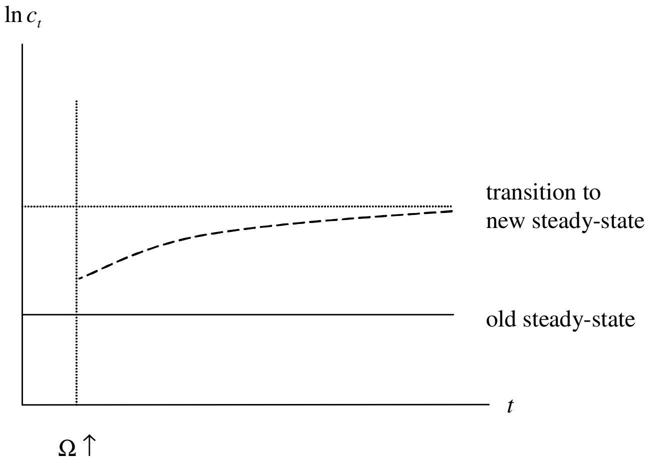

Figure 2. The effects of trade liberalization: the Melitz (2003) "zero growth" case.

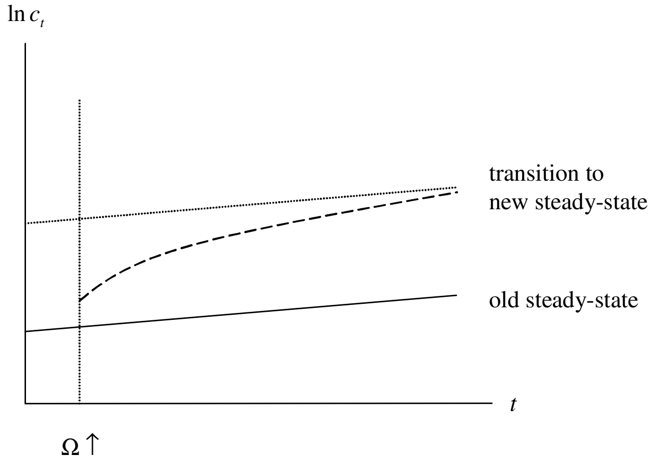

Figure 3. The effects of trade liberalization: the $\phi < \overline{\phi}$ "low growth" case.

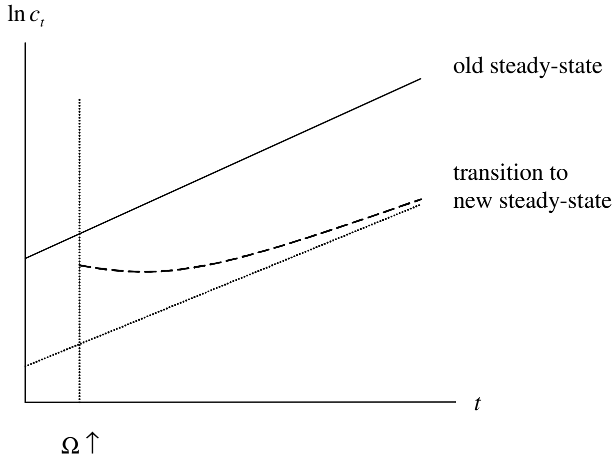

Figure 4. The effects of trade liberalization: the $\phi > \overline{\phi}$ "high growth" case.

Figure 2 illustrates the Melitz (2003) "zero growth" case. The horizontal line represents an old steady-state with a zero rate of productivity growth. When trade liberalization occurs ($\Omega\uparrow$), per capita real consumption c_t jumps up and then there is gradual convergence to a new steady-state with zero productivity growth but higher consumer welfare at each point in time t. Figure 3 illustrates the $\phi < \overline{\phi}$ "low growth" case. The slightly upward sloping line represents an old steady-state equilibrium with a low rate of productivity growth. When trade liberalization occurs ($\Omega \uparrow$), per capita real consumption c_t jumps up and then there is gradual convergence to a new steady-state equilibrium with the same low rate of productivity growth but higher consumer welfare at each point in time t. Figure 4 illustrates the $\phi > \overline{\phi}$ "high growth" case. The significantly upward-sloping line represents an old steady-state equilibrium with a high rate of productivity growth. When trade liberalization occurs ($\Omega \uparrow$), per capita real consumption c_t jumps down and then there is gradual convergence to a new steady-state equilibrium with the same high rate of productivity growth but lower consumer welfare at each point in time t.[9]

The intuition behind Theorem 3 is as follows. When knowledge spillovers are relatively weak ($\phi < \overline{\phi}$) and the steady-state rate of productivity growth $\frac{\dot{c}_t}{c_t}$ given by (32) is relatively low, then the fact that trade liberalization causes the least productive firms to exit (Theorem 1) is more important for productivity growth than the fact that trade liberalization temporarily lowers the variety growth rate $g \equiv \frac{\dot{m}_t}{m_t}$ (Theorem 2). In the "low growth" case, trade liberalization promotes productivity growth in the short run and makes consumers better off in the long run. However, when knowledge spillovers are relatively strong ($\phi > \overline{\phi}$) and the steady-state rate of productivity growth $\frac{\dot{c}_t}{c_t}$ given by (32) is relatively high, then the fact that trade liberalization causes the least productive firms to exit (Theorem 1) is less important for productivity growth than the fact that trade liberalization temporarily lowers the variety growth rate $g \equiv \frac{\dot{m}_t}{m_t}$ (Theorem 2). In the "high growth" case, trade liberalization retards productivity growth in the short run and makes consumers worse off in the long run.

This intuition is helpful for understanding otherwise puzzling results in the earlier literature. In Melitz (2003), trade liberalization has the first effect of causing the least productive firms to exit but the second effect of causing a slowdown in variety growth is absent since

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[9] In Figure 4, the reason why real per capita consumption can immediately drop as a result of trade liberalization is that many firms that had earlier decided not to export under the old trade regime suddenly decide that they do want to export under the new trade regime. With a sudden surge in the measure of firms that incur the fixed cost of entering the export market, real per capita consumption can drop.

there is no variety growth ($g = 0$). Thus the first effect must dominate and Melitz (2003) finds that trade liberalization unambiguously makes consumers better off in the long run. In "the Grossman-Helpman model" in BRN (2006), attention is restricted to the special case where knowledge spillovers are quite strong ($\phi = 1 > \overline{\phi} \equiv 1/(1+\beta)$). Trade liberalization causes a permanent decrease in the variety growth rate ($g \downarrow$) and because this effect must eventually dominate the temporary effect on productivity of the least productive firms exiting ($a_L \downarrow$), BRN find that trade liberalization unambiguously makes consumers worse off in the long run. In our model, the effect of trade liberalization in slowing variety growth is temporary in nature (Theorem 2) and thus the overall effect of trade liberalization on long run welfare can go either way (Theorem 3).

So which of the two cases highlighted in Theorem 3 is empirically more relevant? Alternatively stated, are intertemporal knowledge spillovers in R&D relatively weak or relatively strong? To address this issue, it is helpful to solve for the model's implication for patenting behavior. Assuming that each new variety is associated with a new patent, the patents-per-researcher ratio is given by $\dot{m}_t/L_{It}$. Substituting into (14) using (5) and (25) yields

$$\frac{\dot{m}_t}{L_{It}} = \left(\frac{(\beta - 1)(1+\lambda)^{\phi}}{\beta F_L (1+\Omega)} \right) m_t^{\phi}.$$

Since all terms in the bracketed expression are constant over time in a steady-state equilibrium and m_t increases over time, the patents-per-researcher ratio increases over time if $\phi > 0$ and decreases over time if $\phi < 0$. The patent statistics shed light on which of these cases is more relevant. Kortum (1993, 1997) documents a decreasing patents-per-researcher ratio in a large set of countries. Looking at industry data, Kortum (1993) finds that the patenting per unit of real R&D ratio has declined in all 20 industries for which data could be obtained. Also Jones (2005a) finds evidence of an increasing knowledge burden over time that leads researchers to choose narrower expertise and to compensate for their reduced individual capacities by working in larger teams. All of this evidence leads us to take seriously the possibility that $\phi < 0$, in which case R&D spillovers are definitely in the relative weak range ($\phi < 0 < 1/(1+\beta)$). Based on Theorem 3, R&D spillovers are relatively weak whenever the development of new products becomes more difficult over time.

5 Concluding remarks

This paper explores the implications of trade liberalization when firms are heterogeneous in terms of productivity and there is R&D-driven productivity growth.

Trade liberalization causes the least productive firms to exit and induces more firms to become exporters (Theorem 1). By itself, this market share reallocation from less to more productive firms contributes to productivity growth. But trade liberalization also causes a temporary slowdown in the development of new products (Theorem 2). When more firms choose to incur the fixed costs of becoming exporters, the expected costs of developing profitable new products increase, and it is profit-maximizing for firms to cut back on the development of new products.

The overall effect of trade liberalization on productivity growth depends on the size of intertemporal knowledge spillovers in R&D, the extent to which researchers become more productive in developing new products as the stock of knowledge increases over time (Theorem 3). When these spillovers are relatively weak and the steady-state rate of productivity growth is relatively low, then trade liberalization promotes productivity growth in the short run and makes consumers better off in the long run. However, the reverse holds when knowledge spillovers are relatively strong and the steady-state rate of productivity growth is relatively high. Then trade liberalization retards productivity growth in the short run and makes consumers worse off in the long run.

To keep the analysis as streamlined as possible, this paper has focused on the symmetric country case. It would be interesting to explore how things change when countries differ in terms of resources or trade policies. Then the analysis becomes more complicated because asymmetries in resources or trade policies can produce asymmetric cut-off values for entering local and foreign markets. We have also restricted attention to the case where all innovations are new product varieties. It would be interesting to explore how the model's properties change when firms also do R&D to improve the quality of existing products. Trade liberalization could possibly stimulate firms to devote more resources to new product development in that case. These are possible directions for future research.

References

[1] Aghion, Phillipe and Howitt, Peter (1992), "A Model of Growth Through Creative Destruction", *Econometrica*, 60, 323-351.

[2] Alcala, Fransisco and Ciccone, Antonio (2004), "Trade and Productivity", *Quarterly Journal of Economics*, 199, 613-646.

[3] Aw, Bee Yan, Chung, Sukkyun and Roberts, Mark J. (2000), "Productivity and Turnover in the Export Market: Micro-level Evidence for the Republic of Korea and Taiwan (China)", *World Bank Economic Review*, 14, 65-90.

[4] Baldwin, Richard E. (2005), "Heterogeneous Firms and Trade: Testable and Untestable Properties of the Melitz Model", mimeo, Graduate Institute of International Studies.

[5] Baldwin, Richard E. and Robert-Nicoud, Frederic (2006), "Trade and Growth with Heterogeneous Firms", CEPR Discussion paper No. 5563.

[6] Bartelsman, Eric J. and Doms, Mark (2000), "Understanding Productivity: Lessons from Longitudinal Microdata", *Journal of Economic Literature*, 83, 569-594.

[7] Bernard, Andrew B., Eaton, Jonathan, Jensen, J. Bradford and Kortum, Samuel (2003), "Plants and Productivity in International Trade", *American Economic Review*, 93, 1268-1290.

[8] Bernard, Andrew B. and Jensen, J. Bradford (1999a), "Exceptional Exporter Performance: Cause, Effect or both?", *Journal of International Economics*, 47, 1-25.

[9] Bernard, Andrew B. and Jensen, J. Bradford (1999b), "Exporting and Productivity: Importance of Reallocation", NBER Working Paper No. 7135.

[10] Bernard, Andrew B. and Jensen, J. Bradford (2001), "Why Some Firms Export", NBER Working Paper No. 8349.

[11] Bernard, Andrew B. and Wagner, Joachim (2001), "Export Entry and Exit by German Firms". *Weltwirtschaftliches Archiv - Review of World Economics*, 137, 105-123.

[12] Coe, David T. and Helpman, Elhanan (1995), "International R&D Spillovers", *European Economic Review*, 39, 859-887.

[13] Clerides, Sofronis K., Lach, Saul and Tybout, James R. (1998), "Is Learning by Exporting Important? Micro-dynamic Evidence from Colombia, Mexico and Morocco", *Quarterly Journal of Economics*, 113, 903-947.

[14] Del Gatto, Massimo, Giordano, Mion and Ottaviano, Gianmarco I.P. (2006), "Trade Integration, Firm Selection and the Costs of Non-Europe", CEPR Discussion paper No. 5730.

[15] Dixit, Avinash and Stiglitz, Joseph E. (1977), "Monopolistic Competition and Optimum Product Diversity", *American Economic Review,* 67, 297-308.

[16] Grossman, Gene M. and Helpman, Elhanan (1991), Innovation and Growth in the Global Economy. Cambridge, Mass.: MIT Press.

[17] Helpman, Elhanan and Krugman, Paul R. (1985), Market Structure and Trade, MIT Press.

[18] Helpman, Elhanan, Melitz, Marc J. and Yeaple, Stephen Ross (2004), "Exports vs FDI with heterogeneous firms", *American Economic Review*, 94, 300-317.

[19] Jones, Benjamin F. (2005a), "The Burden of knowledge and the 'Death of the Renaissance Man': Is Innovation Getting Harder?", mimeo, Northwestern University.

[20] Jones, Charles I. (1995a), "Time Series Tests of Endogenous Growth Models", *Quarterly Journal of Economics*, 110, 495-525.

[21] Jones, Charles I. (1995b), "R&D-based Models of Economic Growth", *Journal of Political Economy*, 103, 759-784.

[22] Jones, Charles I. (2005b), "Growth and Ideas", in Aghion, P and Durlauf, S (eds), Handbook of Economic Growth, Elsevier, 11063-1111.

[23] Keller, Wolfgang (2002), "Trade and the Transmission of Technology", *Journal of Economic Growth*, 7, 5-24.

[24] Kortum, Samuel (1993), "Equilibrium R&D and the Patent-R&D Ratio: U.S. Evidence", *American Economic Review*, *Papers and Proceedings,* 83, 450-457.

[25] Kortum, Samuel (1997), "Research, Patenting and Technological Change", *Econometrica,* 65, 1389-1419.

[26] Mankiw, Gregory N., Romer, David and Weil, David (1992), "A Contribution to the Empirics of Economic Growth", *Quarterly Journal of Economics*, 107, 407-438.

[27] Melitz, Marc J. (2003), "The impact of trade on intra-industry reallocations and aggregate industry productivity", *Econometrica,* 71, 1695-1725.

[28] Melitz, Marc J. and Ottaviano, Gianmarco, I.P. (2005), "Market Size, Trade and Productivity", mimeo, Harvard University.

[29] Pavcnik, Nina (2002), "Trade Liberalization , Exit and Productivity Improvements: Evidence from Chilean plants", *The Review of Economic Studies*, 69, 245-276.

[30] Roberts, Mark J. and Tybout, James R. (1997a), "The Decision to Export in Colombia: An empirical Model of Entry with Sunk Costs", *American Economic Review*, 87, 545-564.

[31] Roberts, Mark J. and Tybout, James R. (1997b), "What Makes Exports Boom?", The World Bank

[32] Romer, Paul M. (1990), "Endogenous technical change", *Journal of Political Economy*, 98, 71-102.

[33] Segerstrom, Paul S. (1998), "Endogenous Growth Without Scale Effects", *American Economic Review*, 88, 1290-1310.

[34] Segerstrom, Paul S., Anant, T.C.A. and Dinopoulos, Elias (1990), "A Schumpeterian Model of the Product Life Cycle", *American Economic Review*, 88, 1290-1310.

[35] Tybout, James R. (2003), "Plant- and Firm-level evidence on "New" trade theories", in Choi, E. K. and Harrigan, J. (eds), Handbook of International trade, Blackwell, Oxford.

[36] Yeaple, Stephen Ross (2005), "A simple model of firm heterogeneity, international trade and wages", *Journal of International Economics*, 65, 1-20.