Decomposing Firm-level Sales Variation*

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Abstract

We decompose firm-level sales variation into a firm-specific and firm-destination specific components. We then measure the contribution of firm-specific effects to overall market sales variation. To consistently decompose sales, we use a heterogeneity model of exporting with destination specific, firm-specific, and firm-destination specific latent effects. We show how truncation may bias OLS results. To overcome this bias, we use a Monte-Carlo Estimation-Maximization procedure to estimate firm-specific effects. We find that firm-specific heterogeneity drives less than 31% of sales variation for over half of exported Danish products. When we remove first-time exports from our sample, the median value increase to 40%, implying that firm-destination-specific effects are most important the first year. We conclude that while firm-specific productivity can account for some of the variation, the majority must be explained by firm-destination-specific heterogeneity sources such as market-specific demand.

Keywords:

JEL Codes:

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1 Introduction

There is substantial variation in firm-level export volumes. For example, year 2003 annual shipments for Danish exporters ranged from less than two hundred dollars to more than six-hundred million dollars. Recent theoretical works have attributed this variation to heterogeneity in firm productivities.[1] These theories were motivated by earlier empirical studies that identified differences between firms that do export and firms that do not[2]: on average, exporters produce more, hire more labor, pay higher wages, and exhibit higher productivities as measured by either Total Factor Productivity (TFP) or Value Added per worker. The contrasts between exporters and nonexporters supported the story that productivity and exporting status were linked.

This paper measures how much sales variation can be explained by productivity. We decompose firm sales variation within an destination into a firm-specific component and a firm-destination-specific component. This decomposition attributes all firm heterogeneity to productivity. Since productivity is anchored to the firm, the sales variation explained by firm fixed effects is an upper bound of that explained by productivity heterogeneity.

This paper adds to a small but growing literature examining the destinations to which firms export. The lack of work in the area is due primarily to the dearth of firm-destination specific export observations. Eaton, Kortum, and Kramarz (2004) find that most French firms export to only one destination (the mode being Belgium), and that the entry of French firms into a market accounts for two-thirds of the growth of the French share of market sales. Newer studies show that firms supply domestically for several years before exporting, that they usually begin exporting to one destination country, and that many stop exporting activities soon after they begin[3]. Since productivity is realized before supply to any destination and applies to all destinations, these studies present

[1]Notable examples include Eaton and Kortum (2002), Bernard, Eaton, Jensen, and Kortum (2003), Melitz (2003), Bernard, Jensen, and Schott (2006), and Bernard, Jensen, Redding, and Schott (2007).

[2]Notable works include Aw, Chung, and Roberts (2000), Bernard and Jensen (1995, 1999) and Clerides, Lach, and Tybout (1998).

[3]For example, Eaton, Eslava, Kugler, and Tybout (2007), Damijan, Kostevc, and Polanec (2007), Alvarez, Faruq, and Lopez (2007)

empirical patterns unreconciled by the productivity heterogeneity models. The current study adds to and extends this literature by utilizing a highly disaggregated and detailed dataset - we observe destination-specific shipment values for the universe of Danish exporters in 2001 and 2003. This disaggregation level allows us to identify the firm-specific component and a firm-destination-specific component of an export. We can estimate the contribution of each using our structural model. As it is the standard and flagship productivity heterogeneity model, Melitz (2003) forms the basis for our structural estimation. Since Melitz (2003) does not incorporate destination-specific effects, we incorporate demand heterogeneity à la Nguyen (2009) to account for destination-specific shocks. The resulting model is an amalgam of Melitz (2003) and Nguyen (2009).

Three contemporary studies have goals similar, but not identical, to our own. Eaton, Kortum, and Kramarz (2008) estimates the contribution of firm-specific productivity to sales variance by calibrating an extension of Melitz (2003) to the destination-specific total exports of French firms. Their model incorporates firm-specific productivity shocks drawn from a Pareto distribution and firm-destination-specific taste and cost shocks drawn from lognormal distributions. They calibrate the parameters of their model to match over 1000 distributional moments[4]. Eaton, Kortum and Kramarz (2008) find that firm-specific variation can account for approximately a quarter of the variation of sales across markets.

Kee and Krishna (2008) examine Bangladeshi exports of textiles to the US and EU. They find that a textile firm's market share in EU cannot predict its market share in the US: the correlation between the two is statistically insignificant from zero.

Lawless and Whelan's (2008) study is the closest to ours. They use firm-destination data from a survey of 676 Irish-owned exporters to explain to where and how much firms

[4]For each of the 113 destination countries, Eaton, Kortum, and Kramarz (2007) calibrate their model to match a) the number of French firms selling to that country, b) the number of French firms selling below the fiftieth, seventy-fifth, and ninety-fifth percentile in that country, c) the number of French firms selling in that country and selling below the fiftieth, seventy-fifth,and ninety-fifth percentile in France, d) the number of French firms selling below the fiftieth, seventy-fifth,and ninety-fifth percentile in the France, e) the number of French firms exporting to that country whose ratio of sales in that country to sales in France is below the seventy-fifth of firms selling to that country, and f) the number of simulated firms selling to each possible combination of the seven most popular export destinations.

export. Using OLS regressions, they find the variation in firm-year and country fixed effects accounts for 57 percent of the total variation. By itself, country fixed effects explain 16 percent of the variation, leaving 41 percent of the variation explained by firm-year fixed effects. Our OLS results resemble Lawless and Whelan's, but we show that OLS produces biased results and correct for it.

Compared to these three studies, our study uses a simple, direct method for consistently estimating the contribution of firm-specific effects. It also uses the most detailed dataset of the four. Our dataset covers the universe of Danish firms. It uniquely identifies exports at the six digit product level, a level of disaggregation not available in the three other studies. We can pool observations at differing industry levels, allowing us to compare the contribution of productivity for both broadly defined and narrowly defined industries. Our structural model is simple and leans heavily on Melitz (2003) and Nguyen (2009).

In the next section, we present ana illustrative example of how firm-specific effects may or may not drive firm sales across destinations. Section 3 presents the Danish export data. Next, we present a simple model, based heavily on Melitz (2003) and Nguyen (2009), that shows how truncation biases standard estimation procedures. Section 5 outlines our strategy to overcome this bias. Results and conclusions follow.

2 An illustrative example

To aid the reader in understanding the goal of this paper, we begin with an illustrative example. Suppose Denmark exports to only two destinations: Sweden and Germany. Melitz (2003) predicts that firms that sell to both destinations should have relative revenues that are one-to-one correlated. If a Danish brick firm's sales to Germany are twice the average of all Danish brick firms selling to Germany, this firm must be twice as productive. The same firm's Swedish sales should therefore be twice the average. The one-to-one correlation predicts that the variation in German relative revenues should completely explain the variation in Swedish relative revenues.

We present the results of this test for building bricks, and for cardboard boxes, in Figure 1. It depicts firm revenues to Sweden and Germany for the two products[5]. The revenues are relative to the mean Danish firm revenues of the respective product to the respective destination.

Insert Figure 1 here

The OLS results for cardboard boxes support a weaker interpretation of Melitz: that relative revenues are strongly and positively correlated, and close to one. The slope, although statistically different from one, is still high at 0.84. The variation in German relative revenues explains a little more than half of the variation in Swedish relative revenues.

The OLS results for building bricks do not support the predictions of Melitz. The implied correlation is negative and not statistically different from zero. The $R^2 = 0.08$ suggests that little of the Swedish variation is explained by the German variation. Repeating this procedure for all Danish products exported to both Sweden and Germany, we can test this straight-forward prediction of Melitz (2003). We find a mean and median correlation of 0.18 and 0.12. The mean and median R^2 are both below 5 percent. These initial results suggest that firm-specific characteristics cannot explain much of the cross-destination variation.

For the remainder of the paper, we do not rely on the estimated slope as a measure of the contribution of productivity to relative revenues for several reasons. First, over one third of our estimated slopes are negative, which would imply that productivity does not contribute to destination-specific revenues at all. Second, the correlation does not tell us the contribution of productivity variation to the total variation. For example,

[5] The two products are more precisely "Boxes, cases, crates and similar articles for the conveyance or packaging of goods, of plastics" (CN8 product 39231000) and "Building bricks (excl. those of siliceous fossil meals or similar siliceous earths, and refractory bricks of heading 6902)" (CN8 product 69041000). Sweden and Germany are the two most popular destinations for Danish exporters.

consider Figure 2 below, which presents three graphs of simulated relative revenues in two destinations. All three scatters have fitted slopes of 1, but different R^2 values. If productivity heterogeneity is the sole source of the variation, we should expect to see scatters similar to that in the upper left panel of Figure 2. As the contribution of firm-destination specific heterogeneity rises, the scatters begin to look more like the ones in the upper right and bottom left of Figure 2.Therefore, estimated slopes close to one are misleading confirmations of Melitz (2003).

Insert Figure 2 here

3 Danish Firm Level Data

In Denmark it is required by law that all firms, whether public or private, register with the Central Business Register (CVR) and obtain a unique CVR identifier. The CVR number is used in all public registers and recorded at Statistics Denmark, so any firm's activities can be accurately traced through different registers. One such register is The Danish External Trade Statistics which provides product-level destination-specific export data for all Danish firms. Exports are recorded according to the eight-digit Combined Nomenclature (CN) product code which encompasses approximately 10,000 different product categories. Another register is Account Statistics, which will be used for firm-level productivity calculations. It includes firm size defined as the number of full-time employees, capital stock defined as the value of land, buildings, machines, equipment and inventory, and value added defined as revenues net of input costs and energy consumption.

This study examines 2003 Danish manufacturing exports[6]. Our dataset comprises 145304 firm-destination-product sales observations by 4304 firms in 3331 six-digit Harmonized System (HS6) products to 223 destination countries totaling 189 billion Danish kroner (roughly corresponding to $42 million). Table 1 lists some summary statistics for the number of products, firms and destinations in 2003 by Standard International Trade

[6]We also look at the year 2001, with similar results.

Classification industry (SITC1 = 5, 6, 7, 8):

Insert Table 1 here

Table 1 shows some difference between exporters in Denmark and those in the US (Bernard and Jensen, 1995) or France (Eaton, Kortum, Kramarz, 2004). More than 50% of firms sell their goods to six or more destinations. This is much greater than the median value of a single export destination in the US and French data. Unlike the US or France, Denmark is a small country surrounded by much bigger economies. Approximately 18% of our observations consisted of a firm shipping its product to a single destination. Our empirical strategy cannot identify the firm-specific effect with only a single destination, so these observations must be dropped for our HS6 aggregation level estimations.

While Danish firms exported to more than a handful of destinations, most destinations did not have more than a handful of firms. The median number of firms in an product-destination was only two for the Chemical sector and only three for the materials sector. 23% of our product-destinations contained a single firm. Our empirical strategy cannot identify the destination-specific effect with only a single firm, so these observations must be dropped for our HS6 aggregation level estimations.

It should be noted that while all trade flows with non-EU countries are recorded by customs authorities (and so the coverage rate in the data is close to complete), there is not a similar system in place for intra-EU trade. However, intra-EU trade is recorded through the Intrastat system, where firms are obliged to report trade data on a monthly basis. One source of inaccuracy in this system is that some firms appear not to report data to the system. Also, data on intra-EU trade is censored in a way such that only firms exporting goods with a total annual value exceeding a certain threshold[7] are recorded in the files. No such data limitations exist for trade out of the EU. As a result the coverage rate in the Intrastat system is only in the range 85-90 percent. See Statistics Denmark (2003) for further details.

[7] For 2002, this threshold was DKK 2.5 million corresponding to approximately USD 500,000.

4 Theory

Our model is based on Melitz (2003) and Nguyen (2009). We employ two types of heterogeneity (firm-specific and firm-destination-specific) and consider multiple products and destinations. Ours is a partial equilibrium model identifying the sources of revenue heterogeneity while holding fixed the entry of new products. See Melitz (2003) and Nguyen (2009) for general equilibrium models that evaluate the entering mass of new products by clearing the labor market under a zero-profit condition. Our model's predictions for the variation of revenues across destinations can be collapsed to those of either model. Table 2 in the appendix lists the notation for ease of reference.

4.1 A model of sales variation

The small open economy of Denmark exports goods produced by N products to foreign destinations $j \in J$. For each product $n \in N$, there are W_n Danish firms each producing a unique variety ω. A portion W_{nj} of these firms supply to destination j. For the rest of this section, we focus our attention on a single product and therefore drop the n without loss of generalization. The utility gained in destination j from consuming Danish varieties of this product is represented by u_j :

$$u_j = \sum_{\omega=1}^{W_j} \exp\left(\frac{x_{\omega j}}{\sigma}\right) (q_{\omega j})^{\frac{\sigma-1}{\sigma}}$$

where $\sigma > 1$ is a measure of the substitutability among the different varieties.

The utility function resembles a Dixit-Stiglitz utility function with a demand shifter. The demand shifter $x_{\omega j}$ represents destination j taste[8] for variety ω. Higher $x_{\omega j}$ corresponds to greater demand for that variety relative to other varieties in the same destina-

[8]Nguyen (2009) defines this parameter as "perceived quality." We can also think of it as $\omega's$ popularity or appeal in j.

tion. Destination $j's$ demand for variety ω can be derived as:

$$
\begin{aligned}
q_{\omega j} &= (p_{\omega j})^{-\sigma} \exp(x_{\omega j}) \frac{Y_j}{P_j} \\
P_j &= \sum_{\omega=1}^{W_j} \exp(x_{\omega j}) (p_{\omega j})^{1-\sigma}
\end{aligned}
$$

where $p_{\omega j}$ is the price of ω and Y_j is $j's$ total expenditure on Danish varieties. P_j is the corresponding Chamberlainian price index, which is unaffected by the actions of any single firm.

Firms share similar increasing returns to scale production technologies. Firm $\omega's$ cost $c_{\omega j}$ of supplying $q_{\omega j}$ units of output to destination j is

$$
c_{\omega j}(q_{\omega j}) = f + \exp\left(\frac{b_\omega}{1-\sigma}\right) \tau_j q_{\omega j}.
$$

where f and τ are fixed and variable costs identical to all firms supplying to j. The firm specific $\exp\left(\frac{b_\omega}{1-\sigma}\right)$ is the firm's marginal cost of product that is constant across all destinations. The b_ω term is a normalized measure of $\omega's$ productivity: a higher b translates to a lower marginal cost for the firm across all destinations.

Each firm $\omega \in \{1, ..., m_j\}$ draws its firm-specific productivity b_ω. In addition, each firm draws a firm-destination specific taste parameter $x_{\omega j}$. The two random variables b_ω and $x_{\omega j}$ determine firm $\omega's$ potential sales $r^*_{\omega j}$ in destination j, which is presented in log form:

$$
\begin{aligned}
\ln r^*_{\omega j} &= a_j + b_\omega + x_{\omega j} \qquad (1a) \\
a_j &= \ln\left(\frac{Y_j \tau_j^{1-\sigma}}{P_j}\right)
\end{aligned}
$$

The firm productivity draws $b'_\omega s$ are drawn from exogenous independent normal distributions with product specific mean $\bar{b}$ and variance s_b^2. Likewise, the firm-destination specific taste draws $x'_{\omega j} s$ are drawn from exogenous independent normal distributions with product-specific mean $\bar{x}_j$ variance s_x^2. These normality assumptions are supported by the distribution of domestic revenues of Danish firms presented in Figure 3 and is consistent with previous studies of firm size distribution (Cabral and Mata, 2003) and export selection (Helpman, Melitz, Rubinstein, 2008). The productivity draw and taste draws are constructed to be uncorrelated with one another[9].

Insert Figure 3

The variation in the sales of a firm to a destination is now decomposed into three latent effects: a destination specific effect a_j, a firm-specific effect b_ω, and a firm-destination specific effect $x_{\omega j}$. This study focuses on the contributions of the firm-specific effect and the firm-destination specific effect. We estimate the contribution of the the firm-specific effect to the variance of sales for firms within a destination, controlling for destination-specific effects. That is, this study estimates the statistic

$$Q^2 = \frac{s_b^2}{s_b^2 + s_x^2} \tag{2}$$

for each Danish product exported in 2003.

4.2 Truncation issues

If sales were observed for every firm-destination pair, a simple anova of $\ln r^*_{\omega j}$ on destination and firm-specific effects would consistently decompose the variance, with the residual being attributed to $x_{\omega j}$. However, our dataset is an unbalanced panel where not every firm sells to every destination. The firm-destination sales $r^*_{\omega j}$ is truncated, with the truncation endogenously correlated with a_j, b_ω, and $x_{\omega j}$. In this section, we show how we correct for these sources of bias.

Melitz (2003) suggests that the presence of a firm in a destination is tied to its potential profit in that market. In the current model, firm $\omega' s$ profits π gained from supplying to

[9] Nguyen (2009) discusses the case where firm-destination specific taste draws are positively correlated with another via a latent quality. In this case, the correlation would be attributed to firm-specific productivity. If a firm has higher sales in all destinations, we cannot say whether that is due to higher productivity or higher latent quality.

j are

$$\pi_{\omega j} = \frac{r^*_{\omega j}}{\sigma} - f$$

Profits are positive when $r^*_{\omega j} > c$,where

$$c = \sigma f.$$

and is unknown to the econometrician. Therefore, we cannot observe all $r^*_{\omega j}$. We only observe $r_{\omega j}$, where

$$r_{\omega j} = \begin{cases} r^*_{\omega j} \text{ for } r^*_{\omega j} \geq c \\ 0 \text{ for } r^*_{\omega j} < c \end{cases} \tag{3}$$

Equation (3)given ($1a$) is the standard Type 1 Tobit Model with specific effects described in Honoré and Kyriazidou (2000). In an earlier paper, Honoré (1992) shows that if the specific effects[10] are correlated with the probability of truncation, then the Heckman (1979) 2-step procedure is biased. Honoré's solution to this problem treats the specific effects as nuisance variables and differences them out. In our study, a_j is a nuisance variable, but b_ω is a parameter of interest, so we cannot use Honoré's solution. Instead, we treat a_j as a fixed effect, b_ω as a random effect, and $x_{\omega j}$ as a residual. We then estimate s^2_b and s^2_x using a Monte Carlo Expectation-Maximization Maximum Likelihood Estimation (MCEM) proposed by Walker (1996). Kuhn and Lavielle (2005) show under very general conditions that the MCEM procedure obtains consistent estimates for nonlinear mixed-effects models. Using simulated datasets that resemble our actual dataset, we verify that our MCEM procedure estimates s^2_b and s^2_x consistently.

5 Estimation Strategy

This paper estimates via MCEM the portion of sales variance contributed by firm-specific effects. Using our model given by equation (3), we can derive the distribution of $r_{\omega j}$ given

[10] In our case the specific effects correspond to a_j and φ_ω.

a_j, b_ω, c, and s_x^2:

$$\begin{aligned} \Pr\left(r_{\omega j} = r > c | a_j, b_\omega, c, s_x^2\right) &= \Pr\left(a_j + b_\omega + x_{\omega j} = r\right) \\ &= \frac{1}{s_x}\varphi\left(\frac{r - a_j + b_\omega}{s_x}\right) \qquad (4) \\ \Pr\left(r_{\omega j} = 0 | a_j, b_\omega, c, s_x^2\right) &= \Pr\left(a_j + b_\omega + x_{\omega j} < c\right) \\ &= \Phi\left(\frac{c - a_j + b_\omega}{s_x}\right) \qquad (5) \end{aligned}$$

where $\varphi(\cdot)$ and $\Phi(\cdot)$ are the standard normal pdf and cdf. We will use D to denote the indicator function that takes the value 1 if observed sales $r_{\omega j} = 0$ and 0 if $r_{\omega j} \geq 0$. Combining (4) and (5) with the indicator function D, we derive the conditional probability of $r_{\omega j} = r$:

$$\Pr\left(r_{\omega j} = r | a_j, b_\omega, c, s_x^2\right) = \frac{1}{s_x}\varphi\left(\frac{r - a_j + b_\omega}{s_x}\right)(1 - D) + \Phi\left(\frac{c - a_j + b_\omega}{s_x}\right) D$$

Following Wooldridge (2002), we find L_ω, the joint density of $\vec{r_\omega} = (r_{\omega 1}, r_{\omega 2}, ..., r_{\omega J})$ given b_ω, c, s_x^2 and the vector $\vec{a}_j = (a_1, a_2, ..., a_J)$:

$$L_\omega\left(\vec{r}_\omega | \vec{a}_j, c, b_\omega, s_x^2\right) = \prod_{j=1}^{J}\left(\frac{1}{s_x}\varphi\left(\frac{r_{\omega j} - a_j + b_\omega}{s_x}\right)(1 - D) + \Phi\left(\frac{c - a_j + b_\omega}{s_x}\right) D\right)$$

We treat b_ω as a random effect drawn from a normal with mean 0 and variance s_b^2. Given s_b^2, we can integrate out L_ω's dependence on b_ω. Finally, we sum this integral over all m firms to arrive at our log-likelihood of the unknown parameters given observations $\vec{r} = \{r_{\omega j} | \omega = 1, ..., W; j = 1, ..., J\}$:

$$l\left(\vec{a}_j, c, s_x^2, s_b^2 | \vec{r}\right) = \sum_{\omega=1}^{W} \ln\left(\int_{-\infty}^{\infty} L_\omega\left(\vec{r}_\omega | \vec{a}_j, b_\omega, c, s_x^2\right) \frac{1}{s_b}\varphi\left(\frac{b}{s_b}\right) db\right) \qquad (6)$$

We treat the destination specific effect a_j as a fixed effect. We remain ambivalent as to the underlying distribution, since a_j is not a parameter of interest. We obtain our estimates via MCEM. In each iteration, the b_ω's are integrated out using a forty-point

Gaussian Quadrature (the E-step). The parameters are then obtained by maximizing l (the M-step).

The sample space of $\vec{r}_\omega$ is a function of the unknown parameter c. Zuehkle (2003) suggests estimating c with the minimum order statistic of the untruncated $r_{\omega j}$:

$$\hat{c} = \min\{r_{\omega j} | r_{\omega j} > 0; \omega = 1, ..., W; j = 1, ..., J\} \tag{7}$$

Carson and Sun (2007) proves that $\hat{c}$ converges to c at the rate of $1/W$. They also show that MCEM estimates of the remaining coefficients are asymptotically normal with asymptotic variances identical to the case when c is known. We follow their lead and use $c = \hat{c}$. We then estimate the other parameters in (6) via MCEM, as previously described.

5.1 Monte Carlo Simulation

We verify our procedure's ability to accurately estimate Q^2 under various conditions. We simulate 90 datasets of $(W, J) = (100, 100)$ possible firms and destinations[11] and compare the estimated $\hat{Q}^2$ with the known true$Q^2 = \tilde{Q}^2$. The simulation procedure is outlined as follows:

1. Pick a $\tilde{Q}^2 \in \{0.1, 0.2, .., 0.8, 0.9\}$. Choose s_b^2 and s_x^2 such that $\frac{s_b^2}{s_b^2+s_x^2} = \tilde{Q}^2$ and $s_b^2 + s_x^2 = 4$. Set c equal to 7.5.[12]

2. Draw a_j from a lognormal distribution for each of the J destinations. Draw b_ω from a $n(0, s_b^2)$ for each of the W firms. Draw $x_{\omega j}$ from a $n(0, s_x^2)$ for each of the $J \times W$ observations.

3. Generate $r^*_{\omega j}$ and $r_{\omega j}$ according to equations $(1a)$ and (3).

4. Obtain parameter estimates $\hat{a}_j, \hat{c}, \hat{s}_x^2$ and $\hat{s}_b^2$ via the MCEM procedure described in section 5. Calculate $\hat{Q}^2 = \frac{\hat{s}_b^2}{\hat{s}_b^2+\hat{s}_x^2}$.

[11] We also try $(W, J) = (100, 50)$ wth similar results.

[12] These values were chosen to so that $30 - 70\%$ observations would be truncated.

5. Repeat steps 2, 3, and 4 ten times.

6. Repeat steps 1-5 for all $\tilde{Q}^2 \in \{0.1, 0.2, .., 0.8, 0.9\}$

The results of our simulations are summarized in Figure 4 below. Our estimated $\hat{Q}^2$ tracks the true value $\tilde{Q}^2$ well, with none of the median estimates more than one standard deviation from the true value. There were a handful of outliers that resulted from the MCEM not converging before .

Insert Figure 4 here

6 Estimation Results

We use the MCEM procedure to obtain an estimate Q^2_{MCEM} for contribution of firm-specific effects for each Danish export in 2003[13]. We do this at the HS6 product level. We also perform OLS dummy regressions of destination-mean-differenced observed revenues $\left(\ln r_{\omega j} - \sum_{\omega=1}^{W_j} \ln r_{\omega j}\right)$ on firm fixed-effects. From the OLS regressions, we retrieve the adjusted coefficient of determination $\bar{R}^2$ and estimate Q^2_{ols} by[14]:

$$Q^2_{OLS} = \max\left\{0, \bar{R}^2\right\}$$

Q^2 is defined as a positive number, so we treat negative $\bar{R}^2$ values as estimates of 0 for Q^2. In this section, we compare the MCEM estimates Q^2_{MCEM} to the OLS estimates Q^2_{OLS}.

6.1 Product Level

As noted in the Data Section, there are many HS6 products that contain few firms selling to few destinations. We drop products containing fewer than 25 firm-destination observa-

[13]Results for year 2001 exports were similar to that for 2003. We also exclude small trade flows by deleting observations with a value less than 1,000 DKK. This was to ensure that small value observations did not affect our results. Those results were also similar to the results presented.

[14]We use the adjusted coefficient of determination to avoid small sample bias. Cramer (1987) shows that the unadjusted R^2 is heavily biased upwards for small samples.

tions, products that are exported to only a single destination, and product-destinations categories containing fewer than 5 firms. A total 66488 observations remained, spanning 3790 firms in 480 products to 84 countries and totalling 93 billion Danish kroner. We estimate s_b^2, s_x^2, and consequently Q^2 for each of these 480 products.

Our estimation procedure resulted in mean and median values of 33% and 31% for Q^2_{MCEM} across the 480 HS6 products. This is considerably lower than OLS estimates, which resulted in mean and median values of 43% and 46% for Q^2_{OLS}. For comparison, Lawless and Whelan (2008) obtain an $\bar{R}^2$ of 41% across all Irish exporters. Histograms for the MCEM and OLS estimates are presented in Figure 5 below:

$$\text{Insert Figure 5 here} \tag{8}$$

The histogram of the MCEM estimates in Figure 5 is systematically to the left of that of the OLS estimates. To understand why, we compare the difference between Q^2_{OLS} and Q^2_{MCEM} for each product. The comparison is presented in Figure 6 below:

$$\text{Insert Figure 6 here} \tag{9}$$

As the figure shows, Q^2_{OLS} generally overshoots Q^2_{MCEM}. The overshooting is exacerbated at low values of Q^2_{MCEM}. For products with Q^2_{MCEM} between 10% and 20%, Q^2_{OLS} averages 31%. For products with Q^2_{MCEM} between 20% and 30%, Q^2_{OLS} averages around 41%. This upwards bias shifts the histogram for Q^2_{OLS} to the right.

Q^2_{OLS} actually undershoots Q^2_{MCEM} at values of $Q^2_{MCEM} > 60\%$. Since our simulations showed that Q^2_{MCEM} slightly undershoots the true Q^2 at high values, Q^2_{OLS}'s downward bias is even worse. Q^2_{MCEM} is a more accurate estimator for Q^2 than Q^2_{OLS} across the entire range.

We showed in the previous section that OLS estimates of the contribution of firm-specific effects are generally biased. Our results show that the direction of bias is dependent on the degree to which firm-specific effects affect sales variation. In HS6 products where firm-specific effects do not contribute much to the overal sales variation, an OLS dummy regression overestimates the contribution of the firm-specific effect. In products where firm-specific effects play a large role, an OLS regression underestimates the true contribution.

We show that firm-destination specific effects contribute to over two-thirds of the sales variation in a product-destination market. Our theory suggests that this is due to firm–destination-specific demand variation. Measurement error may be proposed as a possible source of variation. Suppose that is the case; that measurement error is the sole cause of the firm-destination-specific variation. Our sample has a log sales mean of 10.6 and sample log sales variance of 8.4. If measurement error is the cause of two-thirds of that variance, that would imply that an average Danish export recorded at a value of 40,000 kroner has a 68% (1 standard deviation) confidence interval of 4,000 kroner to 420,000 kroner. Our data is customs trade data from which tariff revenues are calculated. It does not seem plausible to have measurement errors that large.

6.2 Industry Level

We also estimate Q^2_{MCEM} and Q^2_{OLS} at the HS2 industry level. We restrict our analysis to industries containing at least 25 firm-destination observations, industries that are exported to at least two destinations, and industry-destinations categories containing at least five firms. With these restrictions, we have 77411 observations spanning 4276 firms in 54 industries exporting to 161 countries, totalling 178 billion kroner.

For the 54 industries, we obtain a median estimates of 32% for Q^2_{MCEM} and 38% for Q^2_{OLS}. This result is in line with our previous estimates at the HS6 product level. For over half of Danish exporting industries, firm-specific effects explain less than a third of total sales variation.

We obtain a mean of 43% for Q^2_{MCEM}, which is higher than the 35% mean obtained for Q^2_{OLS}. This is due to 16 industries having estimates of $Q^2_{MCEM} > 80\%$. Figure 7 show this case.

Insert Figure 7 here

There was no obvious pattern why these industries exhibited higher contributions of firm-specific effects. It is clear that there are several industries in which firm-specific effects play a large role in determining sales. In many other industries, firm-destination-specific effects play the larger role.

6.3 Established Exports

Nguyen (2009) suggests that much of the export sales variation is due to firms testing destinations in order to determine whether they can be successful exporting to that destination. Therefore, firm–destination-specific effects should play a larger role in the first year of exporting and a diminished one in following years. To test that, we restrict our sample to only those firms-product-destination observations in 2003 that were also positive in 2002. That is, only 2003 exports by those firms that exported the same product to the same destination in both 2002 and 2003 were considered. This restriction leaves us with 31242 observations spanning 303 HS6 products, 2491 firms, and 64 countries and totalling 69 billion kroner.

The predictions from Nguyen (2009) were supported by the data. For the 303 established exports in 2003, we obtain mean and median values of 39% and 40% for Q^2_{MCEM} and 49% and 50% for Q^2_{OLS}. These values are a bit higher than those estimates reported for the sample which included first time exports. Firm-destination-specific effects are much more important for the first year of exporting than for the second year of exporting. We present the histogram of results for this sample in figure 8.

Insert Figure 8 here

7 Conclusion

In this paper, we estimate the contribution of firm productivity to the heterogeneity of sales in a destination and find it to be remarkably low. When using firm-specific effects as the broadest interpretation of productivity, we find that it explain less than 31 percent of the sales variation for over half of Danish HS6 products. Our results suggest that firm-specific productivity is not capturing the majority of heterogeneity and is not the primary driver of variation in a market.

We also show that OLS estimates tend to overestimate low contributions and underestimate high contributions. Since the contribution of firm-specific effects are low in most products, OLS regressions tend to overestimate in general. To consistently estimate firm-specific effects, we employ a Monte Carlo Estimation-Maximization strategy used mainly in the Biometrics literature. We suggest that our field adopts this method in future work.

The Melitz (2003) model deftly explains variation between exporters and nonexporters. However, Melitz (2003) is limited to firm-specific differences, and we show that the majority of variation is firm-destination specific. Nguyen (2008) shows how this variation can be generated with a single mechanism involving demand heterogeneity. In it, he presents a model in which firms test destinations and receive firm-destination-specific *perceived quality* draws. Higher *perceived qualities* result in higher sales. Since demands are firm-destination-specific, a firm can have high relative sales in one destination but low relative sales in another. Productivity heterogeneity models cannot generate this sales ranking inversion. He reconciles higher average domestic sales for exporters than for nonexporters by correlating a firm's *perceived qualities* with a firm-specific but unknown-to-the-firm *latent quality.* Since demand is thought of as more uncertain than costs, models incorporating firm-destination-specific demand heterogeneity may better explain the variation of sales.

Our results from restricting the dataset to established exporters also support Nguyen (2009). We show that firm-destination-specific effects are most important the first year of exporting.

Firm-specific heterogeneity does not explain the majority of the sales variation in a destination. An alternative to productivitiy heterogeneity is required to fully explain the patterns of firm-level exports. Firm-destination-specific mechanisms such as Nguyen (2009) should be pursued to explain this rich pattern of firm-level trade.

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Table 1: Table Caption

	Chemicals	**Materials**	**Machinery**	**Misc.**
Number of				
HS6 products	557	1255	85	684
Firms	1045	2463	2643	2544
Destinations	193	194	217	197
Destinations per firm per industry				
mean	23.3	11.3	19.4	17.3
median	12	6	11	11
Firms per destination per industry				
mean	4.4	8.6	16.2	15.8
median	2	3	5	6
Firm sales within a industry-destination (thousand DKK)				
mean mean	3,396	1,085	1,885	1,097
mean median	2,117	582	566	270

Table 2: Notation

Notation	Description
j	A destination country. $j \in J$
ω	The unique variety of a firm
σ	elasticity of substitution between varieties in industry n
$q_{\omega j}$	The quantity of variety ω supplied to j
$p_{\omega j}$	The price of variety ω supplied to j
τ_j	The iceberg trade cost
P_j	The price index in j
$r_{\omega jt}$	The revenue of variety ω in j at t
Y_{jt}	The total expenditure of j
b_ω	The firm-specific productivity of ω
$x_{j\omega}$	The firm-destination-specific demand shock for variety ω in j
s_b^2, s_x^2	The variances of b_ω and $x_{j\omega}$, respectively
Q^2	The theoretical proportion of total variance explained by firm specific effects

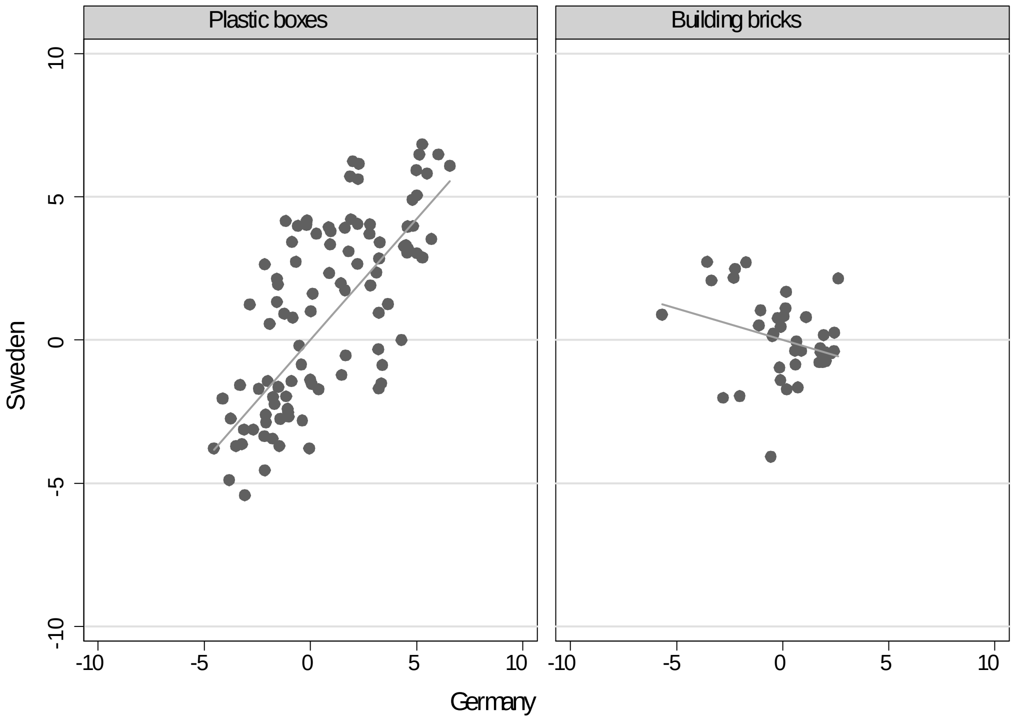

Figure 1: Sales relative to other Danish firms in Sweden and Germany for Danish Exporters of plastic boxes (left panel) and building bricks (right panel). Statistics for the lines with fitted values: Left panel: slope = 0.84, std.err. = 0.08, $R^2 = 0.54$. Right panel: slope = -0.22, std.err. = 0.12, $R^2 = 0.08$.

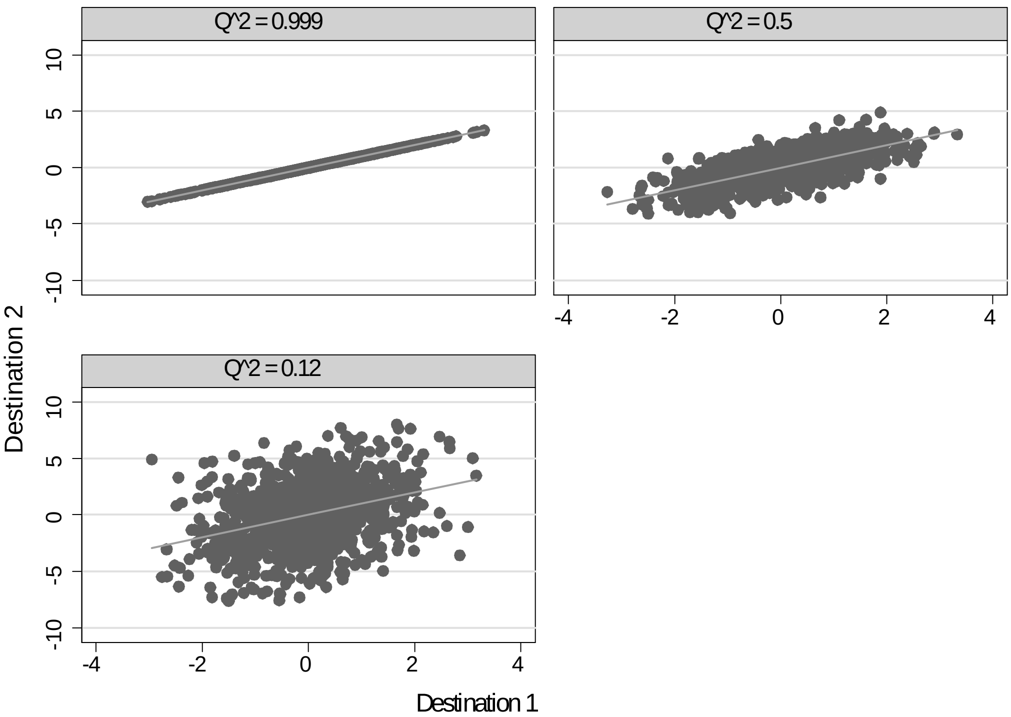

Figure 2: Simulated relative sales in two destinations for varying values of Q_n^2.

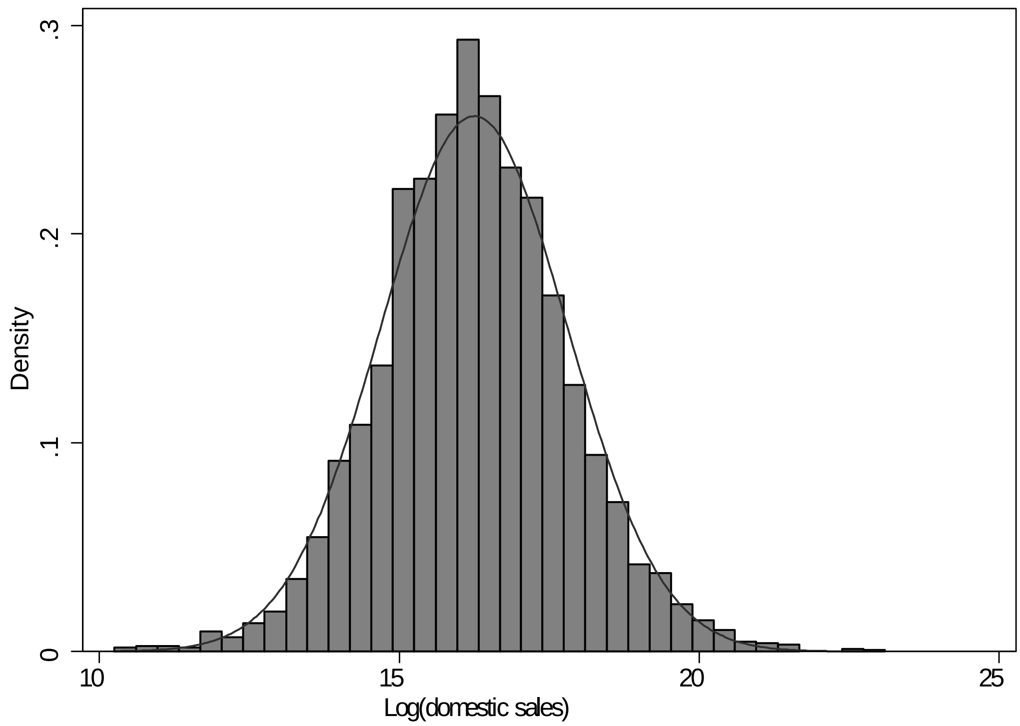

Figure 3: The distribution of log domestic sales for Danish manufacturing firms, 2000.

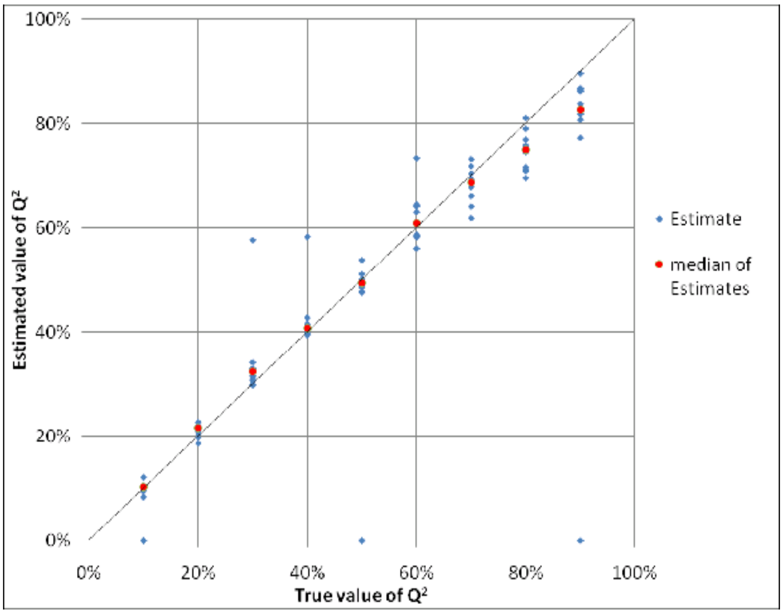

Figure 4: Monte Carlo Simulation results for nine values of Q^2 with ten repetitions each. The EM-MLE estimates are compared to known true values. The circles indicate the median values of the estimates. Estimates lying on the 45^o are exactly equal to the true value.

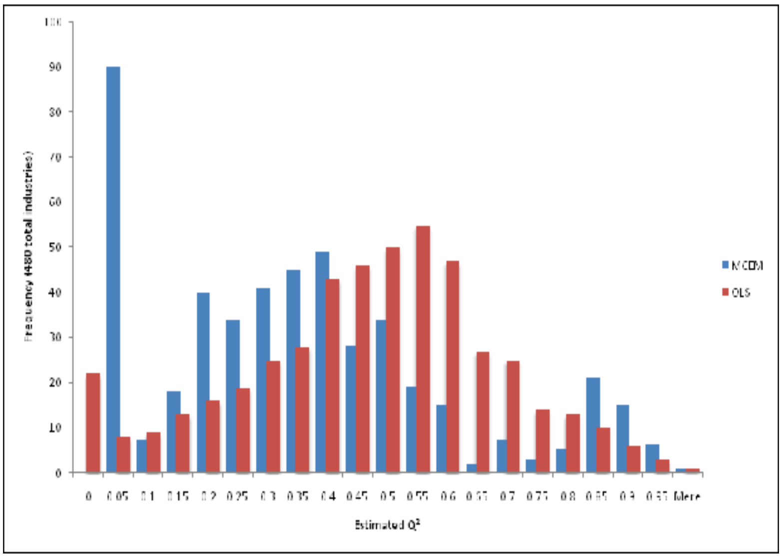

Figure 5: Estimated values for Q^2, the contribution of firm-specific effects, for 2003 Danish exports at the HS6 level.

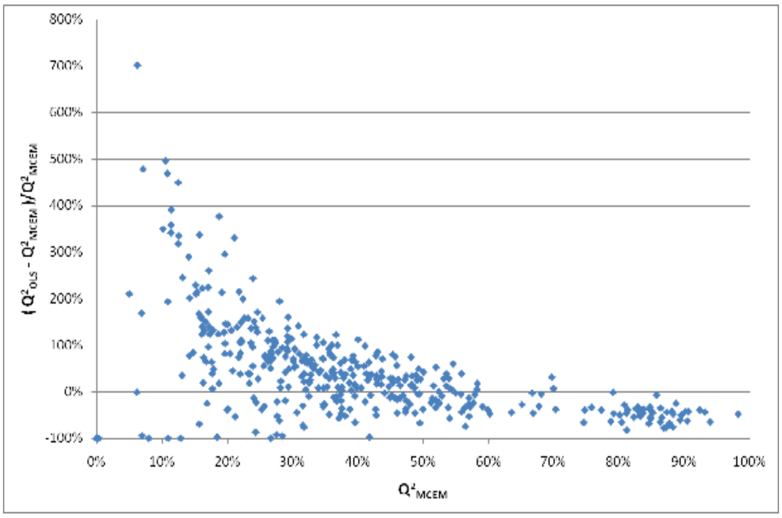

Figure 6: Comparison between MCEM and OLS estimates for the contribution of firm-specific effects.

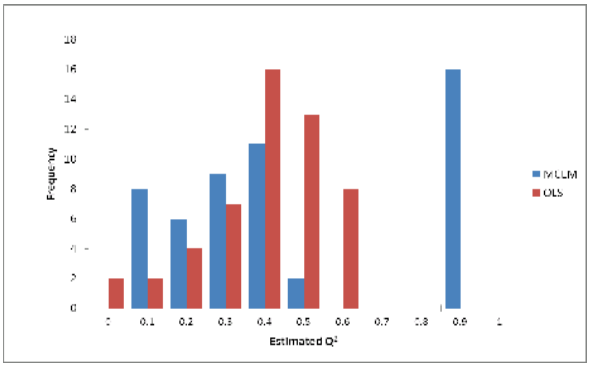

Figure 7: Estimated values for Q^2, the contribution of firm-specific effects, for 2003 Danish exports at the HS2 level.

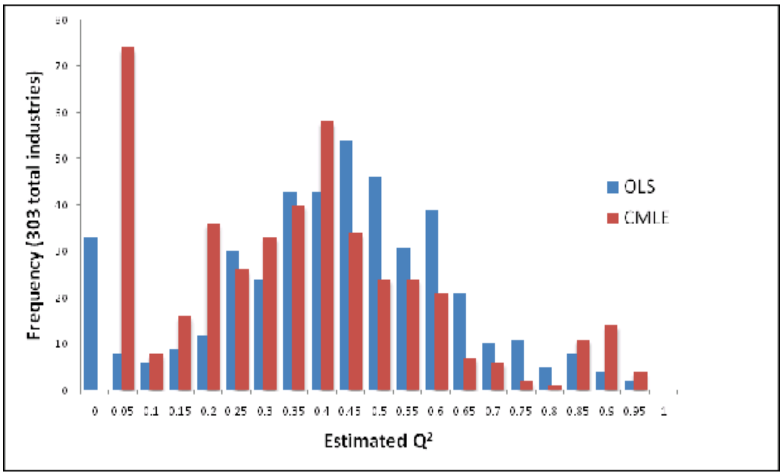

Figure 8: Estimated values for Q^2, the contribution of firm-specific effects, for 2003 Danish exports at the HS6 level. This sample includes only firms that also exported to the same destination in 2002.